

## Math 102 Final Exam Review

1. Compute  $\frac{f(x+h) - f(x)}{h}$  for each of the following functions. Simplify your answers.

a)  $f(x) = x^2 - 4x + 5$

b)  $f(x) = \frac{1}{x}$

c)  $f(x) = -3x^2 - 2x + 1$

d)  $f(x) = \sqrt{x}$

2. Find the domain of each of the following functions.

a)  $f(x) = \frac{x-3}{x^3-9x}$

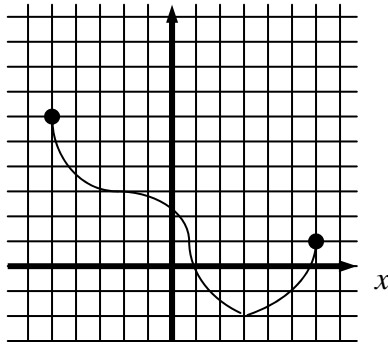
b)  $g(x) = \sqrt{4-3x}$

c)  $h(x) = (x-2)(x+3)$

d)  $f(x) = \frac{\sqrt{x}}{x^2-1}$

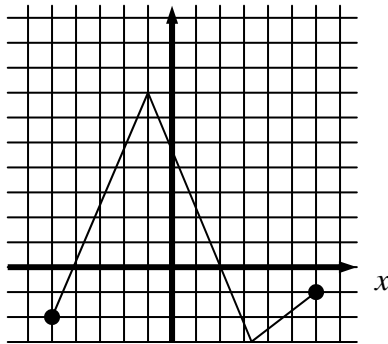
3. Use the graph of the function  $f$  below to answer the following:

a)



- i. What is  $f(-2)$ ?
- ii. What is a local minimum value of  $f$ ?
- iii. What is the range of  $f$ ?
- iv. On what interval is  $f(x)$  decreasing?
- v. What is the average rate of change of  $f$  on the interval  $[-2, 3]$ ?

b)



- i. What is  $f(-1)$ ?
- ii. What is the domain of  $f$ ?
- iii. For how many  $x$  is  $f(x) = -1$ ?
- iv. On what interval is  $f(x)$  increasing?
- v. What is the average rate of change of  $f$  on the interval  $[-1, 6]$ ?

4. Find the average rate of change of

a)  $f(x) = \sqrt[3]{1-x}$  on the interval  $[-7, 9]$

b)  $f(x) = x^2 + 4x$  on the interval  $[1, 5]$

c)  $f(x) = \sqrt{2x-1}$  on the interval  $[1, 5]$

d)  $f(x) = -3x^2 - x$  on the interval  $[5, 6]$

5. Graph each of the following using basic shapes with transformations.

a)  $y = 2 - (x + 3)^2$

b)  $y = \frac{1}{10}(x + 5)^3 - 1$

c)  $y = 10|x - 2| - 5$

d)  $y = -\sqrt{-x} + 3$

6. Construct the following functions.

a) A farmer has 1000 yards of fencing to enclose a rectangular garden. Express the area  $A$  of the rectangle as a function of the width  $x$  of the rectangle. What is the domain of  $A$ ?

b) A wire 20 feet long is to be cut into two pieces. One piece will be shaped as a square and the other piece will be shaped as an equilateral triangle. Express the total area  $A$  enclosed the pieces of wire as a function of the length  $x$  of a side of the equilateral triangle. What is the domain of  $A$ ?

c) A right triangle has one vertex on the graph of  $y = x^2$ , another at the origin, and the third on the positive  $y$ -axis. Express the area  $A$  of the triangle as a function of  $x$ .

d) Two boats leave a dock at the same time. One boat is headed directly east at a constant speed of 35 miles per hour, and the other is headed directly south at a constant speed of 22 miles per hour. Express the distance  $d$  between the boats as a function of time  $t$ .

7. For each of the of the functions below,

i. Determine the slope and  $y$ -intercept of  $f$ .

ii. Use the slope and  $y$ -intercept to graph  $f$ .

iii. Determine the average rate of change of  $f$  on the interval  $[-4, 2]$

iv. Determine whether  $f$  is increasing, decreasing, or constant.

a)  $f(x) = -\frac{3}{2}x + 2$

b)  $f(x) = -3$

c)  $f(x) = -3 + 2x$

d)  $f(x) = \frac{1}{2}x - 3$

8. In 2002, major league baseball signed a labor agreement with the players. In this agreement, any team whose payroll exceeds \$128 million starting in 2005 will have to pay a luxury tax of 22.5% (for first-time offenses). The linear function  $T(p) = 0.225(p - 128)$  describes the luxury tax  $T$  of a team whose payroll is  $p$  (in millions of dollars).

a) What is the implied domain of this function?

b) What is the luxury tax for a team whose payroll is is \$160 million?

c) What is the payroll of a team that pays a luxury tax of \$11.7 million?

9. For each of the quadratic functions below, sketch the graph. Label the vertex, all intercepts, and write the equation of the axis of symmetry.

a)  $f(x) = -2x^2 - 4x - 3$

b)  $f(x) = -x^2 - 4x$

c)  $f(x) = x^2 + 8x + 7$

d)  $f(x) = 3x^2 - 2x + 5$

10. Use each of the following quadratic models to answer the questions.

a) Travel Agency's monthly profit  $P$  (in thousands of dollars) depends on the amount of money  $x$  (in thousands of dollars) spent on advertising per month according to the rule  $P(x) = 7 - 2x(x - 4)$ . What is Paradise's maximum monthly profit?

b) Consider the quadratic model  $h(t) = -16t^2 + 40t + 50$  for the height (in feet),  $h$ , of an object  $t$  seconds after it has been projected straight up into the air. Find the maximum height attained by the object. How much time elapses before the object hits the ground?

11. Construct quadratic models to answer the following.

a) A farmer with 2000 meters of fencing wants to enclose a rectangular plot that borders on a straight highway. If the farmer does not fence the side along the highway, what is the largest area that can be enclosed?

b) Alan is building a garden shaped like a rectangle with a semicircle attached to one short side. If he has 60 feet of fencing to go around it, what dimensions will give him the maximum area in the garden?

12. Graph each polynomial function, label all intercepts, and indicate the end behavior:

a)  $f(x) = x^4 - 2x^3 - 15x^2$

b)  $f(x) = x^5 - 2x^4 - 15x^3$

c)  $f(x) = -x^4 - 4x^3 - 4x^2$

d)  $f(x) = -x^3 - 4x^2 - 4x$

13. Construct a polynomial given the information below:

a) a second degree polynomial with zeros  $-2$  and  $5$  and a constant term of  $-30$

b) a third degree polynomial with zeros  $-1$  and  $2 + 3i$  and a constant term of  $13$ .

c) a fourth degree polynomial with zeros  $2i$  and  $1 - i$  and a leading coefficient of  $2$ .

d) a fourth degree polynomial with zeros  $1 + 2i$  and  $2 - i$  with a constant term of  $100$ .

14. For each of the following,

i) List all possible rational zeros of  $P(x)$ .

ii) Find all roots of  $P(x)$ .

iii) Express  $P(x)$  as a product of linear factors.

a)  $P(x) = 6x^4 + x^3 - 45x^2 + 26x + 24$

b)  $P(x) = 5x^3 - 22x^2 + 18x - 4$

c)  $P(x) = 2x^4 + 5x^3 - x^2 - 3x + 1$

d)  $P(x) = x^5 - 2x^4 + x^3 - 2x^2 - 12x + 24$

15. For each of the following rational functions,

- i) Find the  $x$  and  $y$ -intercepts,
- ii) Find the **equation** of the vertical asymptote.
- iii) Find the **equation** of the horizontal or oblique asymptote.
- iv) Sketch the graph.

a)  $f(x) = \frac{x}{x+1}$

b)  $f(x) = \frac{3x-4}{x-2}$

c)  $f(x) = \frac{x}{x^2-1}$

d)  $f(x) = \frac{x^2-x-12}{x+1}$

16. For each of the following pairs of functions find,

- i.  $(f \circ g)(x)$
- ii. domain of  $(f \circ g)(x)$

a)  $f(x) = \frac{x}{x+1}$ ,  $g(x) = 2x-1$

b)  $f(x) = \frac{3}{x+4}$ ,  $g(x) = \frac{x}{6-5x}$

c)  $f(x) = \frac{x}{x-2}$ ,  $g(x) = \frac{2}{x+5}$

d)  $f(x) = \sqrt{x}$ ,  $g(x) = \sqrt{x+3}$

17. For each of the following functions, find  $f^{-1}(x)$ ,

a)  $f(x) = 3x+5$

b)  $f(x) = \sqrt{3-x}$

c)  $f(x) = \frac{1}{3x-2}$

d)  $f(x) = -x^3+2$

18. Solve the following rational inequalities. (Express answers using interval notation.)

a)  $\frac{x}{x+2} \leq \frac{1}{x}$

b)  $\frac{2x+5}{x+1} \leq 1$

c)  $\frac{9}{x} < x$

d)  $-3 \leq \frac{x+1}{x-3}$

19. Sketch the graph of each of the following.

- a)  $f(x) = -e^{-x}$ . (Label the  $y$ -intercept and the asymptote.)
- b)  $f(x) = 2^{x-3} - 1$  (Label the  $x$ -intercept and the asymptote.)
- c)  $f(x) = -\ln(x+2)$  (Label the  $x$ -intercept and the asymptote.)
- d)  $f(x) = \log_2(-x) + 3$  (Label one point on the graph and the asymptote.)

20. Simplify completely.

- a)  $2\log_5(5x) + \log_5(5x) - 3\log_5 x$
- b)  $4\log_4(2x) - \log_4(x^5) + \log_4(4x)$
- c)  $\log_2 32 + \log_4 64 - \log_{64}(\frac{1}{64})$
- d)  $\log_3 36 + \log_3 5 - \log_3 20$

21. Solve.

- a)  $\log_5(3x-5) = 2$
- b)  $\log_6 x + \log_6(x+5) = 2$
- c)  $\log_{15} x + \log_{15}(x-2) = 1$
- d)  $\log_2 x + \log_2(x-2) = 3$

22. Solve. Give exact answers.

- a)  $2^{x+3} = 5^x$
- b)  $5^{x-1} = 3^x$
- c)  $e^{x-2} = 3^x$
- d)  $\frac{10}{1+e^{-x}} = 2$

23. Solve the following problems involving exponential growth, radioactive decay and interest compounded continuously.

- a) Conservationists tagged 50 black-nosed rabbits in a national forest in the year 2000. In 2003 they tagged 100 black-nosed rabbits in the same range. If the rabbit population follows the law of uninhibited growth, how many rabbits will be in the range in 2008?
- b) The bacteria in a container quadruples every day. If there are initially 100 bacteria, write an equation that models the number of bacteria  $A$  after  $d$  days.
- c) Assume that the half-life of Carbon-14 is 5700 years. Find the age of a wooden axe in which the amount of Carbon-14 is 30% of what it initially had.
- d) Cindy will need \$16,000 in 3 years to return to school and finish her MBA. How much money should she invest now at 9% compounded continuously to have enough money in 3 years?

24. Solve.

$$\text{a) } \begin{cases} .5x + .3y = 2.7 \\ .7x - .2y = 1.3 \end{cases}$$

$$\text{b) } \begin{cases} 6x + y = 7 \\ -24x - 4y = -28 \end{cases}$$

$$\text{c) } \begin{cases} y = 3x + 7 \\ y - 3x = 5 \end{cases}$$

$$\text{d) } \begin{cases} 2x + 18y = -144 \\ 12x + 3y = 81 \end{cases}$$

25. Solve using matrices.

$$\text{a) } \begin{cases} 3x - 2y + z = -7 \\ x + y - 2z = 12 \\ 3x + y - z = 10 \end{cases}$$

$$\text{b) } \begin{cases} 8x + 5y - z = 96 \\ x + 6y - 2z = 44 \\ 9x + y + z = 82 \end{cases}$$

$$\text{c) } \begin{cases} -3x + 7y - z = -8 \\ x + 3y + 8z = 80 \\ -7x + y + z = -38 \end{cases}$$

$$\text{d) } \begin{cases} 6x - y + 2z = 36 \\ -8x - 8z = -88 \\ 4y + z = 15 \end{cases}$$

26. Solve.

$$\text{a) } \begin{cases} x^2 + y^2 = 100 \\ 3x - y = 10 \end{cases}$$

$$\text{b) } \begin{cases} xy = 20 \\ x + y = 9 \end{cases}$$

$$\text{c) } \begin{cases} y = 6x^2 - 5x \\ y = 2x + 3 \end{cases}$$

$$\text{d) } \begin{cases} y = -x^2 + 13 \\ x^2 + y^2 = 25 \end{cases}$$

27. Graph.

$$\text{a) } \begin{cases} 2x + 3y \geq 6 \\ x - y \leq 3 \\ y \leq 2 \end{cases}$$

$$\text{b) } \begin{cases} 4x + 3y \geq 12 \\ x \geq y \end{cases}$$

$$\text{c) } \begin{cases} 2x + 3y \geq 6 \\ x - y \geq 3 \\ y \leq 2 \end{cases}$$

$$\text{d) } \begin{cases} x \geq 0 \\ y \geq 0 \\ x + y \geq 2 \\ x + y \leq 8 \\ 2x + y \leq 10 \end{cases}$$

28. a) Maximize and minimize  $z = 18x + 10y$  subject to the constraints  $x \geq 0$ ,  $y \geq 0$ ,  $2x + 3y \geq 6$ ,  $x \leq 10$ , and  $y \leq 5$
- b) Minimize  $z = 6x + 5y + 17$  subject to the constraints  $x \geq 0$ ,  $y \geq 0$ , and  $x + y \geq 1$
- c) Maximize and minimize  $z = 5x - 15y$  subject to the constraints  $x \geq 0$ ,  $y \geq 0$ ,  $4x + 5y \leq 30$ ,  $4x + 3y \leq 20$ ,  $x \leq 5$ , and  $y \leq 8$
- d) Minimize  $z = 3x + 4y$  subject to the constraints  $x \geq 0$ ,  $y \geq 0$ ,  $2x + 3y \geq 6$ , and  $x + y \leq 8$ .

29. A manufacturer produces two models of bicycles. The times (in hours) required for assembling, painting, and packaging each model are shown in the table.

Process	Hours, model A	Hours, model B
Assembling	2	2.5
Painting	4	1
Packaging	1	0.75

The total times available for assembling, painting, and packaging are 4000 hours, 4800 hours, and 1500 hours, respectively. The profits per unit are \$45 for model A and \$50 for model B. How many of each type should be produced to maximize profit? What is the maximum profit?

30. Prove the following using Mathematical Induction.

- a) For all natural numbers  $n$ ,  $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$
- b) For all natural numbers  $n$ ,  $1 + 3 + 5 + \dots + (2n-1) = n^2$

