

1. Find all six trigonometric functions of θ if $(3, \sqrt{7})$ is on the terminal side of θ .

2. If $\sec\theta = -3$ and the terminal side of θ is in quadrant II, find

a) $\tan\theta$

b) $\csc\theta$

3. Give the **exact** values of:

a) $4\cos\left(-\frac{3\pi}{4}\right)$

b) $\csc\left(\frac{11\pi}{6}\right)$

4. A rectangular swimming pool is three feet deep at the shallow end of the pool. The bottom of the pool has a steady downward drop of 12° . If the pool is 50 feet long, how deep is it at the deep end?

5. In $\triangle ABC$ $C = 90^\circ$, $A = 30^\circ$, and side $b = \sqrt{8}$ cm. Find exact values for the lengths of the following.

a) side a

b) side c

6. Simplify completely: $(\csc x + \cot x)(1 - \cos x)$

7. Find the exact value of:

a) $\cos \left[\tan^{-1} \left(\frac{\sqrt{5}}{2} \right) \right]$

b) $\sin^{-1} \left(\sin \frac{5\pi}{6} \right)$

8. Prove: $(\tan \theta - \sec \theta)^2 = \frac{1 - \sin \theta}{1 + \sin \theta}$

9. Find the **amplitude**, **period**, and **phase shift** of: $y = 5 \sin \left(\frac{\pi}{2} x + \pi \right)$

10. Sketch one complete period of $y = 2 \cos \pi x$

11. If $x = a \tan \theta$, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ and $a > 0$, express $\sqrt{a^2 + x^2}$ in terms of a trig function of θ .

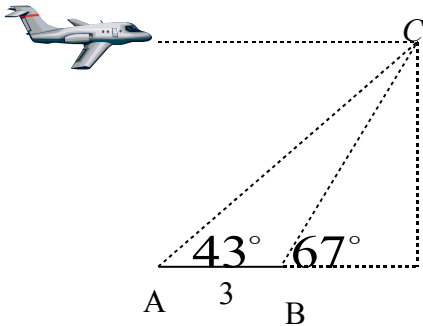
12. Let $\tan A = \frac{3}{2}$ where $\pi \leq A \leq \frac{3\pi}{2}$. Find $\tan \frac{A}{2}$ and simplify your answer.

13. Find all solutions in the interval $[0, 2\pi)$ for: $\tan^2 x \sin x = \sin x$.

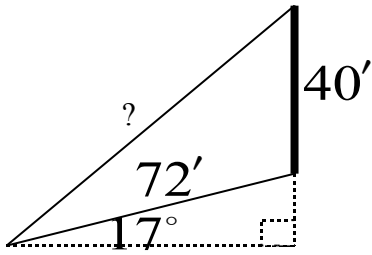
14. Find the resulting equation when the parameter t is eliminated from the equations $x = 3 + 2 \sec t$ and $y = 2 + 4 \tan t$.

15. In triangle ABC, if $\angle A = 67^\circ$, $a = 100$ cm., and $c = 125$ cm, determine if angle C exists. If so, find all possible measures of angle C. If not, explain why.

16. A plane flying in a straight line passes directly over point A on the ground and later directly over point B, which is three miles from A. A few minutes after the plane passes over B, the angle of elevation from A to the plane is 43° and the angle of elevation from B to the plane is 67° . How high is the plane at that moment?



17. A vertical pole 40 feet tall stands on a hillside that makes an angle of 17° with the horizontal. Approximate the shortest length of cable that will reach from the top of the pole to a point 72 feet downhill from the base of the pole.



18. A vector, \mathbf{V} , has magnitude $\|\mathbf{V}\|$ and forms an angle q with the positive x -axis. Find the magnitudes of $\|\mathbf{V}_x\|$ and $\|\mathbf{V}_y\|$ if $\|\mathbf{V}\| = 900$ and $q = 16^\circ$.

19. Find $(-\sqrt{3} + i)^5$. Express your answer in the form $a + bi$.

20. Find the fourth roots of $-8 - 8i\sqrt{3}$.

Math 104 Common Final Exam

Spring 2007

Name: _____

Date: _____ Major: _____

NO GRAPHING CALCULATORS. SHOW ALL YOUR WORK ON THE EXAM. BOX YOUR FINAL ANSWERS.

1 _____ 13 _____

2 _____ 14 _____

3 _____ 15 _____

4 _____ 16 _____

5 _____ 17 _____

6 _____ 18 _____

7 _____ 19 _____

8 _____ 20 _____

9 _____ Total _____

10 _____

11 _____ Grade _____

12 _____

Formulas:

$$\text{Nth roots formula: } w_k = \sqrt[n]{r} \left[\cos\left(\frac{\theta + 2\pi k}{n}\right) + i \sin\left(\frac{\theta + 2\pi k}{n}\right) \right] \quad k = 0, 1, 2, \dots, n-1$$

$$\sin 2A = 2 \sin A \cos A$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A \end{aligned}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \frac{A}{2} = \frac{1 - \cos A}{\sin A} = \frac{\sin A}{1 + \cos A}$$