Math 210 Test #2 Spring 2008

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Instructions: Answer all 11 questions and show your work clearly. There is an extra credit question at the end.

1. (10 pts) a. Write an algebraic expression for each of the following:
i. The number of *yards* in m feet.
To go from FEET to YARDS, you must divide by 3. Why is that? $3ft = 1yd$, $6ft = 2yd$, $9ft = 3yd$, etc. So to find the number of yards in m feet, you take m and divide by 3. I.e., $\frac{m}{3}$ yards.

ii. The total value *in cents* of d dimes, n nickels, and p pennies.
$10d + 5n + p$ cents

iii. The cost *in dollars* of renting x DVDs if it costs \$25 for a membership card (that allows you to rent) and then \$3 for each DVD rented.
$25 + 3x$ dollars

b. Use the identity $a^2 - b^2 = (a + b)(a - b)$ to calculate $77^2 - 23^2$ (without working out 77^2 or 23^2). Show work.
$77^2 - 23^2 = (77 + 23)(77 - 23) = 100 \cdot 54 = 5400$

2. (6 pts) Solve the following word problem using a bar diagram or algebra–your choice! Show all work.
John has twice as much money as Peter and Peter has \$60 more than David. If all three boys have a total of \$600, how much money did they each have at the beginning?
Using ALGEBRA:
Let x be David's amount of money in dollars. Then Peter has $x + 60$ dollars, and since John has twice as much money as Peter, John has $2(x + 60) = 2x + 120$ (need to distribute here). We also know that all three boys have a total of \$600, so we know that all three amounts add up to 600. I.e.,
$x + x + 60 + 2x + 120 = 600$
$4x + 180 = 600$
$4x = 420$
$x = 105$ So David has \$105. Then Peter has $\$105 + 60 = \$165$, and John has $\$2 \cdot 165 = \$330$

3. (15 pts) a. Test the single number 76,593,440 for divisibility by each of the following. Give a brief reason for each.

by 2 Yes, the last digit is 0 (even).
by 3 No. The sum of the digits is 38, and 3 does NOT divide 38.
by 4 Yes since $4|40$ (the last two digits)
by 5 Yes since the last digit is a 0.
by 6 No since not divisible by 3.
by 8 Yes since $8|440$, the number represented by the last three digits.
by 9 No since 9 does not divide 38 (the sum of the digits)
by 10 Yes since the last digit is a 0
by 11 Yes since the sum of the digits in even powers of 10 spots is: $6+9+4+0=19$, and the sum of digits in odd powers of 10 spots is $7+5+3+4=19$. Notice that $11|(19-19)$ or $11|0$ since $0=11\cdot 0$.

b. List ALL digits that make each of the following true.
i. $3|37\diamondsuit 4$
We need 3 to divide $14+\diamondsuit$ (this is the sum of the digits). Digits that make this true are 1, 4, 7.

ii. $4|9083\diamondsuit$
Digits that make this true are 2 (since $4|32$) and 6 (since $4|36$).

4. (6 pts) a. In the **Test for Primeness**, what primes need to be checked for the numbers 197 and 253? (The list is the same for both.) Note that $13^2 = 169$ & $17^2 = 289$.
In the TEST FOR PRIMENESS, need to check divisibility by all primes p satisfying $p^2 \leq number$ 2, 3, 5, 7, 11, 13. (17 is too big since $17^2 = 189$ which is greater than both 197 and 253).

In (b) or (c), if you say "PRIME", clearly show how you are doing the **Test for Primeness**.
b. Is 197 prime or composite? Why?
$2 \nmid 197$, $3 \nmid 197$, $5 \nmid 197$, $7 \nmid 197$, $11 \nmid 197$, $13 \nmid 197$ SO 197 is PRIME!
c. Is 253 prime or composite? Why?
$2 \nmid 253$, $3 \nmid 253$, $5 \nmid 253$, $7 \nmid 253$, BUT $11|253$, so that means that 253 is COMPOSITE (in fact $253 = 11 \cdot 23$).

5. (8 pts) Let $a = 2^3 \cdot 5^2 \cdot 7^3 \cdot 13^6 \cdot 19^2$ and $b = 5^2 \cdot 7^5 \cdot 11 \cdot 13^4$. Find:
(a) $GCF(a, b) = 5^2 \cdot 7^3 \cdot 13^4$
To find GCF, take the product of the COMMON primes raised to the smaller or equal powers.

(b) $LCM(a, b) = 2^3 \cdot 5^2 \cdot 7^5 \cdot 11 \cdot 13^6 \cdot 19^2$
To find LCM, take the product of each prime appearing raised to the higher or equal powers.
(please do not multiply it out–you may leave your answer in prime factored form)

6. (8 pts) a. Find GCF(5040, 208) using the **EUCLIDEAN ALGORITHM**. Show all work (be careful when doing your divisions).
First divide 5040 by 208: It goes in 24 whole times with remainder 48
Second divide 208 by 48: It goes in 4 whole times with remainder 16
Third divide 48 by 16: It goes in 3 whole times with no remainder.
This tells us: $GCF(5040, 208) = GCF(208, 48) = GCF(48, 16) = GCF(16, 0) = 16$

b. Using your answer to (a), find LCM(5040, 208) using an appropriate formula. (It is NOT necessary to work this out–you can leave as is).
Formula $LCM(a, b) = \frac{a \cdot b}{GCF(a,b)}$
So, $LCM(5040, 208) = \frac{5040 \cdot 208}{16} = 65,520$

7. (15 pts) a. Fill in the Blanks:
i. A number is divisible by 18 if and only if it is divisibly by both ——-2——— and ——-9——- (fill in two appropriate numbers).
Need to fill in two numbers that are RELATIVELY PRIME whole product equals 18. Notice that $2 \cdot 9 = 18$, and GCF(2,9)=1.

ii. If a divides b, then GCF(a,b)=——a—— and LCM(a,b)=——-b———-
First think of a specific example where $a|b$; for example 4|12. Now GCF(4,12)=4 and LCM(4,12)=12
Why is this true in general? a is the biggest number that divides a. Since we're given that a divides b, we must have that GCF(a,b)=a. b is the smallest (positive) multiple of itself ($b = b \cdot 1$). Since we're given b is a multiple of a, we must have LCM(a,b)=b.

iii. If a and b are relatively prime, then $LCM(a, b) = \frac{a \cdot b}{GCF(a,b)} = \frac{a \cdot b}{1} = a \cdot b$

iv. If $n = 2^4 \cdot 5^3 \cdot 11^2 \cdot 23$, then the prime factorization of $n^2 = (2^4 \cdot 5^3 \cdot 11^2 \cdot 23)^2 = 2^8 \cdot 5^6 \cdot 11^4 \cdot 23^2$

b. Answer **TRUE** or **FALSE**. IF FALSE, provide a counter-example.
——-**FALSE**—— i. If a is even and b is odd, then GCF(a,b)=1
For ex, 3 is odd and 6 is even, but GCF(3,6)=3 (NOT 1)

——-**FALSE**—— ii. If $4|a$ and $6|a$, then $24|a$
For ex, 4|12 and 6|12, then $24 \not| 12$. Note that 4 and 6 are not relatively prime.

——-**TRUE**—— iii. If a number is divisible by 36, then it is also divisible by 2, 3, 4, 6, 9, 12, and 18.

——-**FALSE**—— iv. If 8 does not divide a and 8 does not divide b, then 8 does not divide $a - b$.
For ex, $8 \not| 9$ and $8 \not| 1$, but $8|(9-1)$.

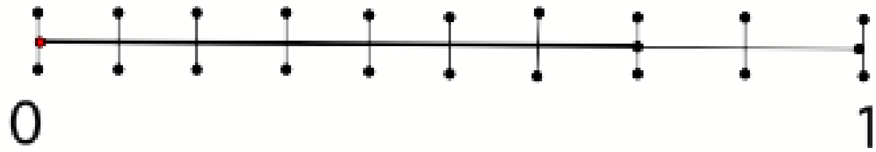
8. (6 pts) Two bike riders ride around a circular path. Tony completes one lap around in 75 minutes. Sam completes one lap around in 90 minutes.
a. If they both start at the same place and the same time and go in the same direction, after how many minutes will they first meet again at the starting place? Show work.
Need to find LCM(75,90).
$75 = 3 \cdot 5^2$
$90 = 2 \cdot 3^2 \cdot 5$
$LCM(75, 90) = 2 \cdot 3^2 \cdot 5^2 = 450$ minutes.

b. At this time, how many laps around has Tony completed?
$450 \div 75 = 6$, so Tony completed 6 laps.

How many laps around has Sam completed?
$450 \div 90 = 5$, so Sam completed 5 laps.

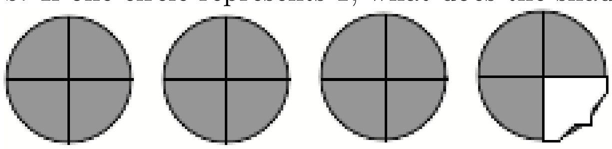
9. (15 pts)

a. What fraction is represented by the shaded portion? (Imagine that the segment is cut into equal-sized pieces.)



$\frac{7}{9}$

b. If one circle represents 1, what does the shaded portion represent? Give your answer two ways:



i. as a mixed number: $3\frac{3}{4}$

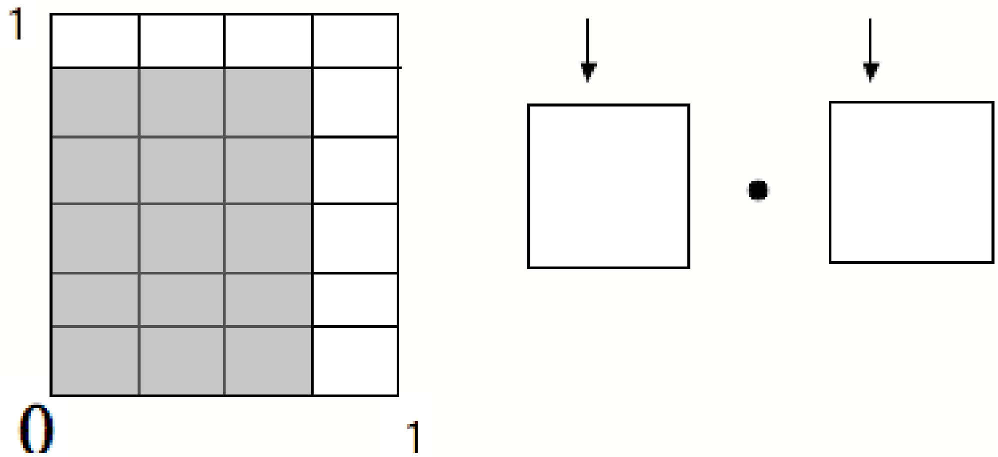
ii. as an improper fraction: $\frac{15}{4}$

c. Which is LARGER: $\frac{5}{9}$ or $\frac{4}{7}$? Give one reason why.

Reason 1: put them over a common denominator. $\frac{4}{7} = \frac{36}{63}$ and $\frac{5}{9} = \frac{35}{63}$. Since $36 > 35$, we can conclude that $\frac{4}{7} > \frac{5}{9}$.

Reason 2: $\frac{4}{7} > \frac{5}{9}$ since $4 \cdot 9 > 7 \cdot 5$.

d. What PRODUCT is represented by the following area model?
Simply fill in the two fractions being multiplied.



The given picture represents $\frac{5}{6}$ of $\frac{3}{4}$ or the product $\frac{5}{6} \cdot \frac{3}{4}$.

e. Dick ate $\frac{3}{4}$ of a box of chocolates. Jane ate $\frac{1}{6}$ of the amount Dick ate. (You may want to use a bar diagram.)

i. What fraction of the box did they eat together?

Dick ate $\frac{3}{4}$. Jane ate $\frac{1}{6}$ of $\frac{3}{4}$ or $\frac{1}{6} \cdot \frac{3}{4} = \frac{1}{8}$ of the box. Together they ate $\frac{3}{4} + \frac{1}{8} = \frac{6}{8} + \frac{1}{8} = \frac{7}{8}$ of the box.

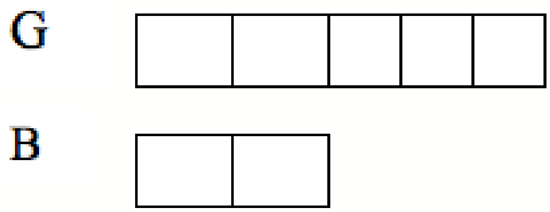
ii. What fraction of the box remains?

$1 - \frac{7}{8} = \frac{1}{8}$.

f. Find $5\frac{1}{3} \div 2\frac{2}{5}$ and express your answer as a mixed number.
$5\frac{1}{3} \div 2\frac{2}{5} = \frac{16}{3} \div \frac{12}{5} = \frac{16}{3} \cdot \frac{5}{12} = \frac{4}{3} \cdot \frac{5}{3} = \frac{20}{9} = 2\frac{2}{9}$

10. (5 pts) Solve by either using a bar diagram or using algebra–your choice! Show all work.
Borders sold $\frac{5}{8}$ of its Harry Potter books. If 273 copies remain how many were there originally?
Subce $\frac{5}{8}$ of the books are sold, that means that $\frac{3}{8}$ of the books remain. So if we let 1 unit be one eighth (one of the boxes in our bar diagram, then 3 units=273. So 1 unit is 91. There are eight units originally, so 8 units=$8 \times 91 = 728$ books.

11. (6 pts) Give a Teacher's Solution using a bar diagram. Show work. Do not use algebra.
$\frac{2}{7}$ of the children in a drama club are boys.
There are 36 more girls than boys.
How many children are in the drama club altogether?



So $3units = 36$, so $1unit = 12$. To find the total number of children, note that there are 7 units in all, so $7units = 7 \cdot 12 = 84$. There are 84 children altogether.

EXTRA CREDIT: 1. (7 pts) Larry takes $\frac{5}{6}$ of an hour to mow $\frac{2}{5}$ of the lawn. At that rate, how long will it take Larry to mow the entire lawn?
$\frac{5}{6}$ hour $= \frac{5}{6}$(60 minutes)$=$ 50 minutes. So if Larry can mow $\frac{2}{5}$ of the lawn in 50 mins, that means (dividing by 2) that he can mow $\frac{1}{5}$ of the lawn in 25 mins. Thus, since the entire lawn is $\frac{5}{5}$ of the lawn (multiplying by 5), it will take Larry $25 \cdot 5 = 125$ mins OR 2 hrs 5 mins.

2. (3 pts) ON THE BACK–John is serving cupcakes at a school party. If he arranges the cupcakes in groups of 2, 3, 4, 5, or 6, he always has exactly one cupcake left over. What is the smallest number of cupcakes he could have?
Since John has one leftover when dividing by 2, 3, 4, 5, or 6, we know that one less than this number is a multiple of all of these numbers. So we find the LCM. Look at their prime factorizations:
$2 = 2$
$3 = 3$
$4 = 2^2$
$5 = 5$
$6 = 2 \cdot 3$
$LCM = 2^2 \cdot 3 \cdot 5 = 60$. And one more than 60 is 61. Thus the smallest number of cupcakes he could have is 61.