Math 104 Test #3 Solutions Fall 2007

B. Noble

1. (7 pts) Solve for θ: $\tan\theta - 2\cos\theta\tan\theta = 0$ if $0^\circ \leq \theta < 360^\circ$.
Factor out $\tan\theta$: $\tan\theta(1 - 2\cos\theta) = 0$, we set each factor equal to zero using the zero-product property: $\tan\theta = 0$ and $1 - 2\cos\theta = 0$. Solving for θ, we obtain:
$\tan\theta = 0$ implies $\theta = 0^\circ$ or $\theta = 180^\circ$
Similarly, $1 - 2\cos\theta = 0$ implies $\cos\theta = \frac{1}{2}$, so $\theta = 60^\circ$ or $\theta = 300^\circ$

2. (8 pts) Solve for x: $\cos 2x + 3\sin x - 2 = 0$ if $0 \leq x < 2\pi$. Write your answers in exact values only.
First, we use a double-angle formula for cosine so that the whole equation is in terms of sine only:
$1 - 2\sin^2 x + 3\sin x - 2 = 0$, which simplifies to $(2\sin x - 1)(\sin x - 1) = 0$.
Setting each factor to zero, we obtain $\sin x = \frac{1}{2}$ and $\sin x - 1 = 0$. The first gives us $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$, the second gives us $x = \frac{\pi}{2}$.

3. (10 pts) Find all degree solutions to $\sin 2x\cos x + \cos 2x\sin x = \frac{1}{\sqrt{2}}$
Use the sum formula for sine to simplify the above equation to $\sin 3x = \frac{1}{\sqrt{2}}$.
If we let $\alpha = 3x$, then $\alpha = 45^\circ$ or $\alpha = 135^\circ$.
So $3x = 45^\circ + 360^\circ k$ or $3x = 135^\circ + 360^\circ k$ (because we want to find all degree solutions).
Solving for x (by dividing each equation by 3), we obtain that $x = 15^\circ + 120^\circ$ or $x = 45^\circ + 120^\circ k$

4. (4 pts each) Eliminate the parameter t in the following parametric equations:
a. $x = 3\cos t - 3$ and $y = 3\sin t + 1$
Solving for sine and cosine, we obtain $\cos t = \frac{x+3}{3}$ and $\sin t = \frac{y-1}{3}$. Using the Pythagorean Identity $sin^2 x + \cos^2 x = 1$, we have that $(\frac{x+3}{3})^2 + (\frac{y-1}{3})^2 = 1$
b. $x = 5\cos t$, $y = 7\cos t$
Here, we can eliminate the parameter by eliminating cosine, namely by noticing that $\cos t - \cot t = 0$. Thus, $\frac{x}{5} - \frac{y}{7} = 0$

5. (10 pts) If $A = 32^\circ$, $B = 70^\circ$, and $a = 3.8$ inches, find the missing parts of $\triangle ABC$.
Using the Law of Sines, $b = \frac{3.8\sin 70^\circ}{\sin 32^\circ} \approx 6.7384$ inches
Similarly, $c = \frac{3.8\sin 78^\circ}{\sin 32^\circ} \approx 7.0142$ inches.

6. (10 pts) Find two triangles for which $A = 51°$, $a = 6.5$ ft, and $b = 7.9$ ft. Using the Law of Sines, $\sin B = \frac{7.9 \sin 51°}{6.5} \approx 0.94453$, so $B = 71°$ or $B = 109°$ (both have reference angle of 71° and sine is positive in QI and QII). These two different possible values of B determine the two different triangles we are looking for.
First, let's look at the triangle determined by the value $B = 71°$.
If we know A and B, then we can find that $C = 58°$. Now, using the Law of Sines, we find that $c = \frac{6.5 \sin 58°}{\sin 51°} \approx 7.093$ feet.
Lastly, let's look at the triangle determined by the value $B = 109°$.
So $C = 20°$, and using the Law of Sines, $c = \frac{6.5 \sin 20°}{\sin 51°} \approx 2.86$ feet.

7. (10 pts) Solve $\triangle ABC$ if $a = 34$ km, $b = 20$ km, and $c = 18$ km.
We'll use the Law of Cosines first to find at least one of the angles in the triangle: $\cos A = \frac{34^2 - 20^2 - 18^2}{-1(20)(18)} = -0.6$, so $A \approx 127°$
Now that we know at least one angle, we can use the Law of Sines to find angle C: $\sin C = \frac{18 \sin 127°}{34} \approx 0.4228$, so $C \approx 25°$
And since we now know two out of the three angles, we can easily find that $B = 28°$.

8. (7 pts) Find the area of $\triangle ABC$ if $a = 24$ in, $b = 14$ in, and $c = 18$ in.

To use Heron's Formula, we first find s: $s = 28$, and $A = \sqrt{15680} \approx 125.2 in^2$.

9. (2 pts each) If $\mathbf{U} = 5\mathbf{i} + 12\mathbf{j}$ and $\mathbf{V} = -4\mathbf{i} + \mathbf{j}$, find:
a. Write $\mathbf{U}$ and $\mathbf{V}$ in component form, $< a, b >$.
$\mathbf{U} =< 5, 12 >$ and $\mathbf{V} =< -4, 1 >$
b. $\mathbf{U} + \mathbf{V} =< 1, 13 >$
c. $4\mathbf{U} - 5\mathbf{V} =< 0, 43 >$
d. $|\mathbf{U} + \mathbf{V}| = \sqrt{170}$
e. $\mathbf{U} \bullet \mathbf{V} = -18$

10. Let $\mathbf{U} = 13\mathbf{i} - 8\mathbf{j}$ and $\mathbf{V} = 2\mathbf{i} + 11\mathbf{j}$. Find:
a. (2 pts) $|\mathbf{U}| = \sqrt{233}$
b. (2 pts) $|\mathbf{V}| = 5\sqrt{5}$
c. (2 pts) $\mathbf{U} \bullet \mathbf{V} = -62$
d. (4 pts) Find the angle θ between $\mathbf{U}$ and $\mathbf{V}$ to the nearest tenth of a degree.
$\cos\theta = \frac{-62}{\sqrt{233}5\sqrt{5}} \approx -0.3632944$, so $\theta \approx 111.3°$.

11. (10 pts) Jorge pulls Alice and Tigran in a wagon by exerting a force of 25 pounds on the handle at an angle of $30°$ with the horizontal. How much work is done by Mark in pulling the wagon 300 feet?
This was done in class. The answer is $3750\sqrt{3}$ ft-lbs.

BONUS QUESTION: (7 pts)
A vector $\mathbf{F}$ is in standard position, and makes an angle of $285°$ with the positive x-axis. Its magnitude is 30. Write $\mathbf{F}$ in component form $< a, b >$ and in vector component form $a\mathbf{i} + b\mathbf{j}$.