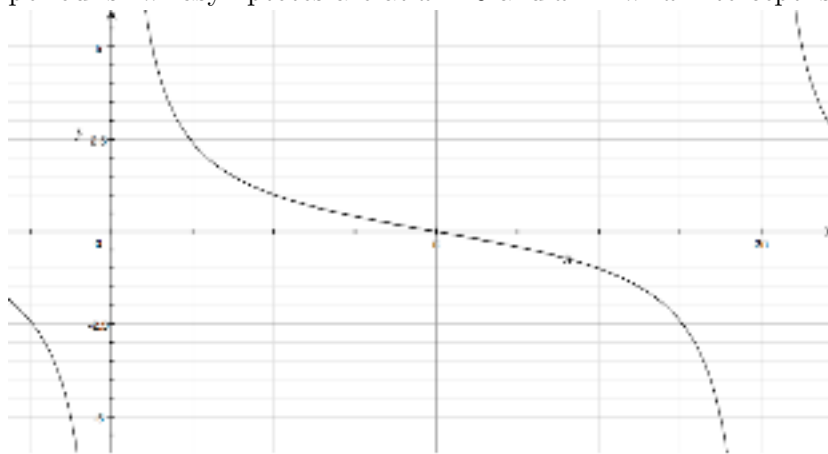


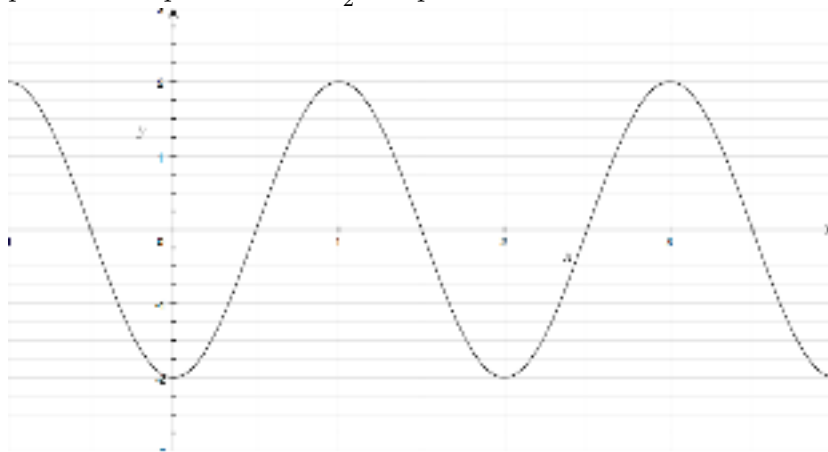
Math 104 Test #2 Fall 2007 October 31, 2007

DIRECTIONS for #s 1-5: For each graph, label the axes accurately and if applicable, identify the period, amplitude, phase shift, or asymptotes. **Neatness counts!** Make sure your graphs are easy to read.

1. (6 pts) Graph one complete cycle of $y = \cot(\frac{1}{2}x)$.
period is 2π . asymptotes are at $x = 0$ and $x = 2\pi$. x -intercept is at $(\pi, 0)$

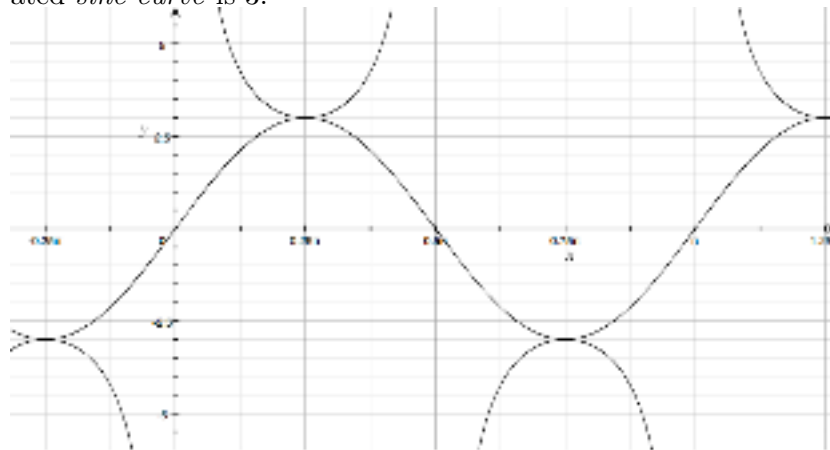


2. (6 pts) Graph one complete cycle of $y = 2 \sin(\pi x - \frac{\pi}{2})$
period is 2. phase shift is $\frac{1}{2}$. amplitude is 2.



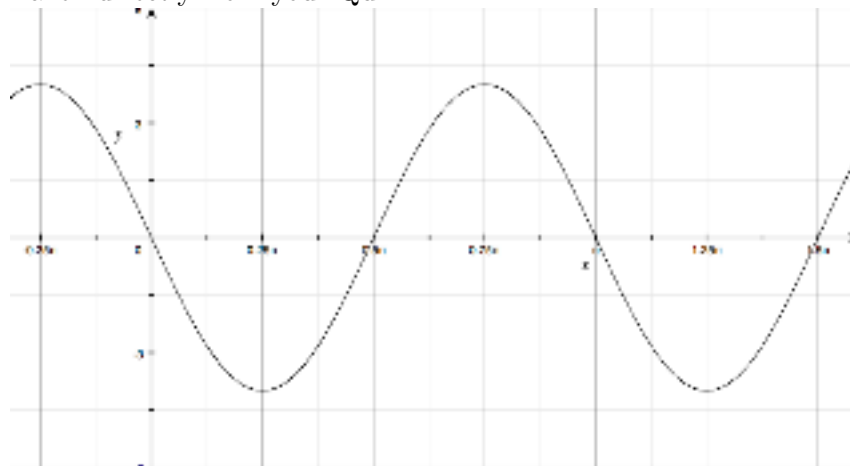
3. (6 pts) Graph one complete cycle of $y = 3 \csc(2x)$ by first graphing the appropriate sine or cosine curve and then using the reciprocal relationships.

period is π . asymptotes are $x = 0$, $x = \frac{\pi}{2}$, $x = \pi$. amplitude of the associated *sine curve* is 3.

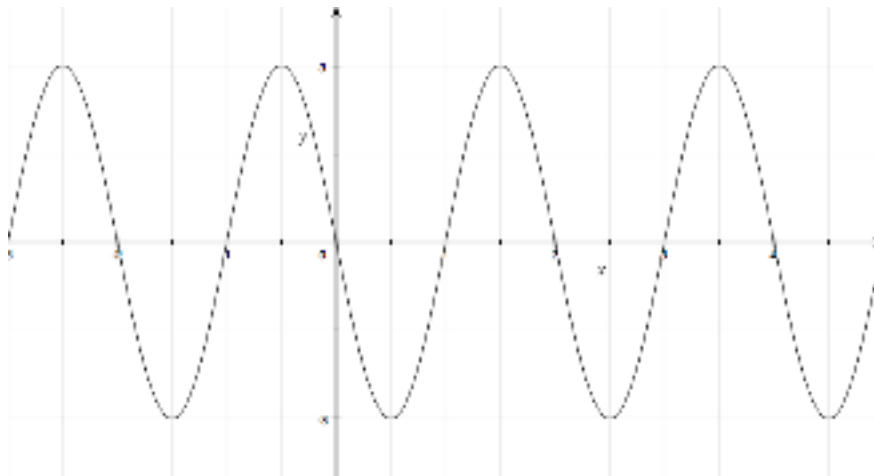


4. (6 pts) Graph the equation $y = -4 \cos(2x - \frac{\pi}{2})$, on the interval $-\frac{\pi}{4} \leq x \leq \frac{3\pi}{2}$.

Taken directly from your Quiz.



5. (6 pts) Graph $y = 3 \sin(-\pi x)$ on the interval $-2 \leq x \leq 4$.
 First use the fact that sine is odd to simplify: $y = -3 \sin(\pi x)$. period is 2.
 amplitude is 3. it is reflected in the x -axis.



6. (6 pts each) Each graph below is one complete cycle of the graph of an equation containing a trigonometric function. In each case, find an equation to match the graph.

- a. $y = 0.5 + 0.5 \sin\left(\frac{\pi}{2}x\right)$
 b. $y = 2 \sin\left(\frac{1}{2}x + \frac{\pi}{2}\right)$

For the remainder of the exam, please box in your final answers.

7. (4 pts) Simplify $7|\csc \theta|$ if $\theta = \arctan \frac{x}{7}$ for some real number x .
 $\frac{7\sqrt{x^2+49}}{|x|}$.

8. (2 pts each) Evaluate without using a calculator. Show all necessary work to prove that you did not merely plug this into your calculator. If no appropriate work is shown, it will not receive any credit.

- a. $\cos(\arctan \frac{3}{4}) = \frac{4}{5}$
 b. $\sin(\arctan \frac{4}{3}) = \frac{4}{5}$
 c. $\sin(\arccos(-\frac{2}{3})) = \frac{\sqrt{5}}{3}$
 d. $\cot(\arccos(-\frac{2}{3})) = \frac{-2}{\sqrt{5}}$

9. Prove the following identities, showing all necessary steps:

a. (6 pts) $\cos^4 \phi = \frac{1}{4} + \frac{\cos 2\phi}{2} + \frac{\cos^2 2\phi}{4}$
 $\cos^4 \phi = (\cos \phi)^4 = \left(\sqrt{\frac{1+\cos 2\phi}{2}}\right)^4 = \left(\frac{1+\cos 2\phi}{2}\right)^2 = \left(\frac{1}{2}\right)^2(1 + \cos 2\phi)^2 = \frac{1}{4}(1 + 2 \cos 2\phi + \cos^2 2\phi) = \frac{1}{4} + \frac{\cos 2\phi}{2} + \frac{\cos^2 2\phi}{4}$

b. (4 pts) $\frac{\cos x}{1+\sin x} - \frac{1-\sin x}{\cos x} = 0$
 $\frac{\cos x}{1+\sin x} - \frac{1-\sin x}{\cos x} = \frac{\cos^2 x - (1-\sin x)(1+\sin x)}{(1+\sin x)\cos x} = \frac{\cos^2 x - (1-\sin^2 x)}{(1+\sin x)\cos x} = \frac{\cos^2 x - \cos^2 x}{(1+\sin x)\cos x} = 0$

10. (3 pts each) Find the *exact* values for the following expressions, and simplify:

a. $\sin \frac{7\pi}{12} = \sin(\frac{\pi}{3} + \frac{\pi}{4}) = \sin \frac{\pi}{3} \cos \frac{\pi}{4} + \sin \frac{\pi}{4} \cos \frac{\pi}{3} = \frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$

b. $\cos 105^\circ = \cos(45^\circ + 60^\circ) = \cos 45^\circ \cos 60^\circ - \sin 45^\circ \sin 60^\circ = \frac{\sqrt{2}}{2} \frac{1}{2} - \frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$

c. Use a half-angle formula to find $\cos 30^\circ$

First note that the the angle 30° terminates in QI, so cosine is POSITIVE.

$$\cos 30^\circ = \sqrt{\frac{1+\cos 60^\circ}{2}} = \sqrt{\frac{1+\frac{1}{2}}{2}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

d. $\sin 15^\circ$

You can do this problem by using either the half-angle identity or the difference formula for sine. The answer may look different, but it really is the same!

$$\sin 15^\circ = \sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \sin 30^\circ \cos 45^\circ = \frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} - \frac{1}{2} \frac{\sqrt{2}}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

12. (5 pts) Let $\cos \theta = \frac{4}{5}$ with θ in QIV and $\cos \alpha = -\frac{12}{13}$ with α in QIII. Compute $\cos(\theta + \alpha)$.

First, we find $\sin \theta = -\frac{3}{5}$ and $\sin \alpha = -\frac{5}{13}$ (draw the appropriate triangles).

Then, we use the sum formula for cosine: $\cos(\theta + \alpha) = \cos \theta \cos \alpha - \sin \alpha \sin \theta = \frac{4}{5} \frac{-12}{13} - \frac{-3}{5} \frac{-5}{13} = \frac{-63}{65}$

13. (3 pts) Express as a single trig function and then simplify:

$$\cos \frac{\pi}{12} \cos \frac{5\pi}{12} - \sin \frac{\pi}{12} \sin \frac{5\pi}{12} = \cos(\frac{\pi}{12} + \frac{5\pi}{12}) = \cos \frac{6\pi}{12} = \cos \frac{\pi}{2} = 0$$

14. (6 pts) If $x = 5 \tan \theta$, write the expression $\frac{\theta}{2} - \frac{\sin 2\theta}{4}$ in terms of just x .

If $x = 5 \tan \theta$, then we can solve for $\theta = \arctan \frac{x}{5}$.

$$\frac{\theta}{2} - \frac{\sin 2\theta}{4} = \frac{\theta}{2} - \frac{2 \sin \theta \cos \theta}{4} = \frac{\theta}{2} - \frac{\sin \theta \cos \theta}{2} = \frac{1}{2}(\arctan \frac{x}{5} - \frac{5x}{x^2+25})$$

Draw the right triangle with $\tan \theta = \frac{x}{5}$, and find sine and cosine there to obtain the above result.

15. (4 pts each) If $\sin \beta = -\frac{1}{\sqrt{5}}$ with $180^\circ \leq \beta \leq 270^\circ$, find:

a. $\cos 2\beta = 1 - 2(\frac{-1}{\sqrt{5}})^2 = \frac{3}{5}$

b. $\cos \frac{\beta}{2}$

First note that since $180^\circ \leq \beta \leq 270^\circ$, that means that $90^\circ \leq \frac{\beta}{2} \leq 135^\circ$, so $\frac{\beta}{2}$ is in QII. Thus $\cos \frac{\beta}{2}$ is negative.

To use the half-angle formula for cosine, we need to first find $\cos \beta = \frac{-2}{\sqrt{5}}$.

$$\cos \frac{\beta}{2} = -\sqrt{\frac{1+\cos \beta}{2}} = -\sqrt{\frac{1+\frac{-2}{\sqrt{5}}}{2}} = -\sqrt{\frac{\sqrt{5}-2}{2\sqrt{5}}}$$

16. Simplify each of the following using an identity. Leave your answer as a trig function.

a. (1 pt) $2 \sin 15^\circ \cos 15^\circ = \sin 30^\circ$

b. (1 pt) $1 - 2 \sin^2 75^\circ = \cos 150^\circ$