Math 104 Test #1 SOLUTIONS Fall 2007
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You must work all of your problems on the exam. **Show ALL of your work** and box in your final answers. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks, class notes, crib sheets, or *graphing* calculators are not permitted. **Don't forget units!**

1. a. (7 pts) Find all six trigonometric functions of θ if $(2, -\sqrt{12})$ is on the terminal side of θ.
$x = 2$, $y = -\sqrt{12} = -2\sqrt{3}$, so $r = 4$ by the Pythagorean Theorem.
$\sin\theta = \frac{-\sqrt{12}}{4} = \frac{-2\sqrt{3}}{4} = \frac{-\sqrt{3}}{2}$
$\cos\theta = \frac{1}{2}$
$\tan\theta = -\sqrt{3}$
$\csc\theta = \frac{-2}{\sqrt{3}}$
$\sec\theta = 2$
$\cot\theta = \frac{-1}{\sqrt{3}}$

b. (4 pts) Find $\sin\theta$ and $\sec\theta$ if $\tan\theta = \frac{12}{5}$ and θ terminates in quadrant III. Rationalize all denominators.
Since θ is in QIII, $x < 0$ and $y < 0$. So $x = -5$ and $y = -12$, and then by the Pythagorean Theorem, $r = 13$.
$\sin\theta = \frac{-12}{13}$
$\cos\theta = \frac{-5}{13}$, so then $\sec\theta = \frac{-13}{5}$.

2. (4 pts) a. Find $\cos\theta$ and $\sin\theta$ if the terminal side of θ lies along the line $y = -x$ in quadrant IV.
Take any point on the line $y = -x$, for example take $(1, -1)$. Then $x = 1$, $y = -1$, and by the Pythagorean Theorem, $r = \sqrt{2}$.
$\sin\theta = \frac{-1}{\sqrt{2}}$
$\cos\theta = \frac{1}{\sqrt{2}}$.
b. (2 pts) Find θ. *Hint: you don't need to use an inverse function.*
Since this is a $45° - 45° - 90°$ triangle, we know that the reference angle, $\hat{\theta} = 45°$. Since θ is in QIII, $\theta = 360° - 45° = 315°$.

3. a. (4 pts) Show that $\sec\theta\cot\theta - \sin\theta = \frac{\cos^2\theta}{\sin\theta}$ by transforming the left side into the right side.
$\sec\theta\cot\theta - \sin\theta = \frac{1}{\cos\theta}\frac{\cos\theta}{\sin\theta} - \sin\theta = \frac{1}{\sin\theta} - \frac{\sin^2\theta}{\sin\theta} = \frac{1-\sin^2\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta}$

b. (4 pts) Use identity substitutions to simplify as much as possible:
$\sqrt[3]{-(\frac{\sin\theta}{\csc\theta}+\frac{\cos\theta}{\sec\theta})} = \sqrt[3]{-(\sin^2\theta+\cos^2\theta)} = \sqrt[3]{-1} = -1$

4. Find
a. (2 pts) $\cot 71°15\prime$ to 3 decimal places, by first converting to decimal degrees.
$71°15\prime = 71.25°$, then $\cot 71.25° = \frac{1}{\tan 71.25°} = -0.633$
b. (2 pts) θ if θ is acute and $\csc\theta = 1.1547$ to 3 decimal places.
$\csc\theta = \frac{1}{\sin\theta} = 1.1547$. Taking the reciprocal of both sides, we have $\sin\theta = \frac{1}{1.1547}$. Then take the inverse sine (also known as arcsine) of both sides, giving us $\theta = 60.000°$.

5. (5 pts) Simplify the expression $\sqrt{x^2+4}$ as much as possible after substituting $2\tan\theta$ for x.
$\sqrt{(2\tan\theta)^2+4} = \sqrt{4\tan^2\theta+4} = \sqrt{4(\tan^2\theta+1)} = \sqrt{4\sec^2\theta} = 2|\sec\theta|$

6. a. (1 pt) In which quadrant will θ lie if $\csc\theta < 0$ and $\cos\theta > 0$?
QIV
b. (1 pt) In which quadrant will θ lie if $\tan\theta < 0$ and $\sec\theta > 0$?
QIV

7. (2 pts each)
a. Convert to radians. Leave your answer in terms of π.
i. $-120° = \frac{-2\pi}{3}$
ii. $250° = \frac{25\pi}{18}$
b. Convert to degrees:
i. $\frac{4\pi}{3} = 240°$
ii. $\frac{7\pi}{12} = 105°$

8. In $\triangle ABC$, $C = 90°$, $B = 60°$, and $b = 15$cm. Find EXACT answers for each of the following, and simplify as much as possible:
a. (3 pts) Side c is $10\sqrt{3}$ cm.
b. (3 pts) Side a is $5\sqrt{3}$ cm.

9. In $\triangle ABC$, $C = 90°$, $a = 24.3$cm, and $c = 48.1$cm. Draw the triangle and then find each of the following:
a. (3 pts) Side b is 41.5 cm (using 3 significant figures)
b. (3 pts) Angle A is 30.3°

c. (3 pts) Angle B is $59.7°$

10. (6 pts) A bullet is fired into the air with an inital velocity of 800 feet per second at an angle of $62°$ from the horizontal. Find the magnitudes of the horizontal and vertical vector components of the velocity vector to two decimal places.

Horizontal magnitude is $800 \cos 62° = 375.58$fps.
Vertical magnitude is $800 \sin 62° = 706.36$fps.

11. (8 pts) Two people decide to find the height of a tree. They position themselves 25 feet apart in line with, and on the same side of, the tree. If they find that the angles of elevation from the ground where they are standing to the top of the tree are $65°$ and $44°$, how tall is the tree to the nearest foot?

From out diagram we obtain two equations:
The first is $\tan 44° = \frac{h}{x+25}$
The second is $\tan 65° = \frac{h}{x}$.
Since we have two equations and two unknowns, we can solve for x & h, but we are only concerned with finding h.
After doing some algebra, we obtain $h = 44$ft (rounded to the nearest foot).

12. (1 pt each) For each of the following, find the reference angle $\hat{\theta}$. Answer in the same units (radians/degrees) you were given.
a. $\theta = -30°$
$\hat{\theta} = 30°$
b. $\theta = \frac{7\pi}{6}$
$\hat{\theta} = \frac{\pi}{6}$
c. $\theta = 390°$
$\hat{\theta} = 30°$
d. $\theta = \frac{5\pi}{3}$
$\hat{\theta} = \frac{\pi}{3}$

13. (2 pts) a. On the unit circle, what ordered pair (x, y) corresponds to the angle $\frac{5\pi}{3}$ radians?
From 12d, we already know that $\hat{\theta} = \frac{\pi}{3}$. Combining this with the fact that θ terminates in QIV, we have $(\frac{1}{2}, \frac{-\sqrt{3}}{2})$.

Give EXACT values for each of the following:
b. (1 pt) $\sin\frac{5\pi}{3} = \frac{-\sqrt{3}}{2}$
c. (1 pt) $\cos\frac{5\pi}{3} = \frac{1}{2}$
d. (1 pt) $\csc\frac{5\pi}{3} = \frac{-2}{\sqrt{3}}$
e. (1 pt) $\cot\frac{5\pi}{3} = \frac{-1}{\sqrt{3}}$

14. (5 pts) Show that cotangent is an odd function.
$\cot(-\theta) = \frac{\cos(-\theta)}{\sin(-\theta)} = \frac{\cos\theta}{-\sin\theta} = -\frac{\cos\theta}{\sin\theta} = -\cot\theta$

15. (2 pts each) Use a calculator to find θ to two decimal places if θ is between 0° and 360°, and
a. $\cos\theta = -0.4772$ with θ in quadrant III.
$\hat{\theta} = \arccos 0.4772 = 66.4973°$. Since θ is in QIII, then $\theta = 180° + \hat{\theta} = 241.50°$.
b. $\sec\theta = 1.545$ with θ in quadrant IV.
$\sec\theta = \frac{1}{\cos\theta} = 1.545$, so taking the reciprocal of both sides, we have $\cos\theta = \frac{1}{1.545}$. Then $\hat{\theta} = \arccos\frac{1}{1.545} = 49.6655°$. Since θ is in QIV, $\theta = 360° - \hat{\theta} = 310.33°$.

16. For a circle of diameter $d = 12$ft, with central angle $\theta = 60°$, find:
Since $d = 12$ft, that means that $r = 6$ft. Converting θ to radians, we have $\theta = \frac{\pi}{3}$.
a. (4 pts) The arc length, s.
We use our formula $s = r\theta = (6)(\frac{\pi}{3}) = 2\pi$ feet.
b. (4 pts) The area of the sector, A.
We use our formula $A = \frac{r^2\theta}{2} = 6\pi$ square feet.

BONUS QUESTION:
(5 pts) If $\sin\theta = \frac{1}{a}$ with θ in quadrant III, find $\cos\theta$, $\csc\theta$, and $\cot\theta$.
Since θ is in QIII, we have that $y = -1$, $r = |a|$, and by the Pythagorean Theorem $x = -\sqrt{a^2 - 1}$ (since θ terminates in QIII, both x and y are negative).
$\cos\theta = -\frac{\sqrt{a^2-1}}{|a|}$
$\csc\theta = a$
$\cot\theta = \sqrt{a^2 - 1}$