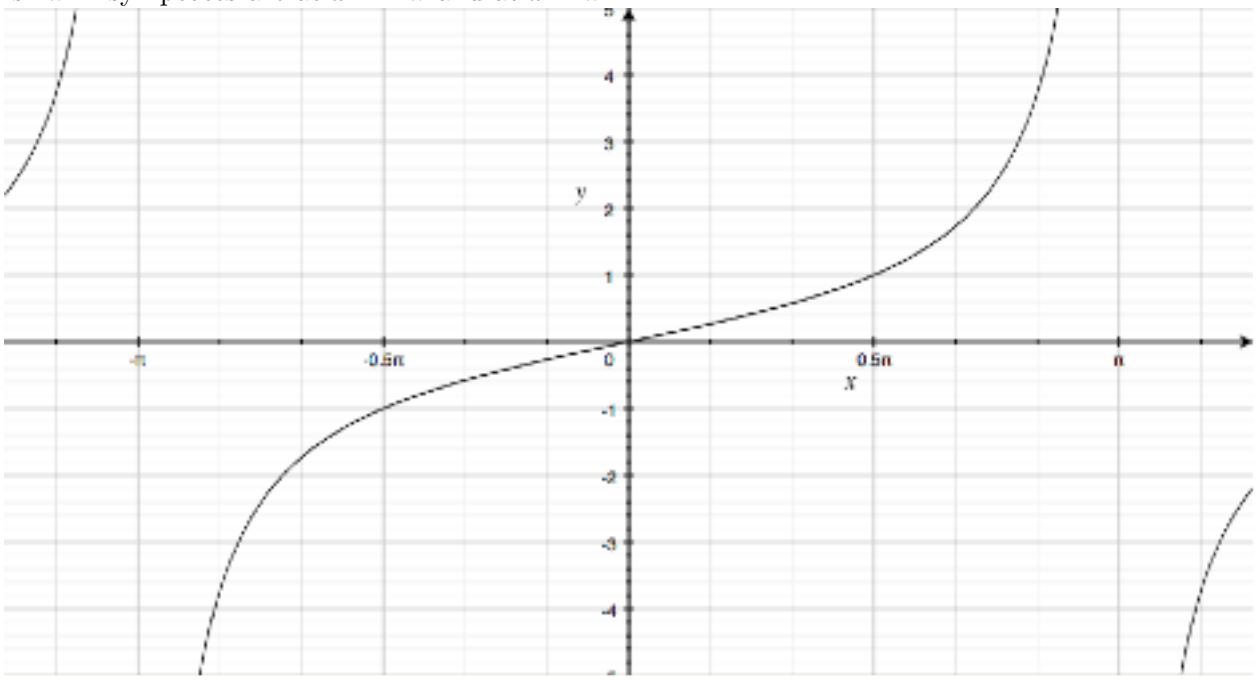
Math 104 Quiz Solutions #3 Fall 2007

B Noble

DIRECTIONS: For each graph, label the axes accurately and if applicable, identify the period, amplitude, phase shift, or asymptotes. **Neatness counts!** Make sure your graphs are easy to read.

Please note that with my graphing utility, I usually wasn't able to exactly graph 1 cycle, but it is approximately 1 cycle.

1. (5 pts) Graph one complete cycle of $y = \tan \frac{1}{2}x$.
For tangent and cotangent, *period* $= \frac{\pi}{B}$. In this case, $B = \frac{1}{2}$, so the period is 2π. Asymptotes are at $x = -\pi$ and at $x = \pi$.



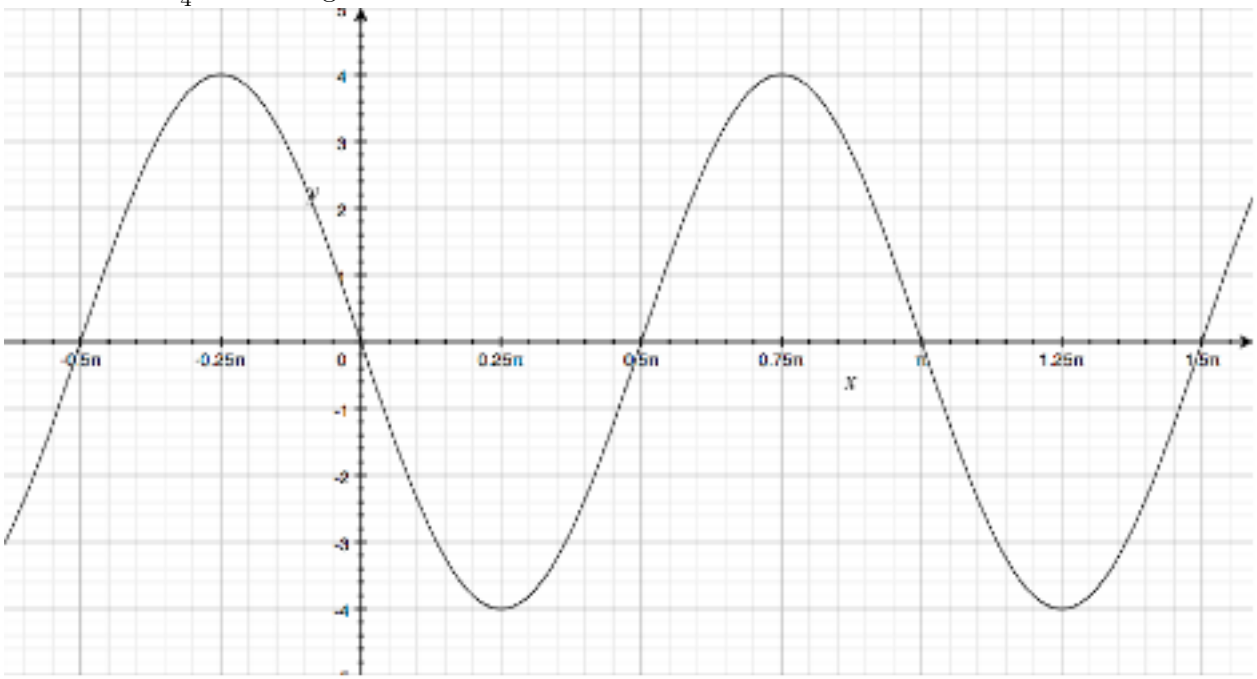
2. (5 pts) Graph the equation $y = -4\cos(2x - \frac{\pi}{2})$, on the interval $-\frac{\pi}{4} \leq x \leq \frac{3\pi}{2}$.

Amplitude is $|-4| = 4$

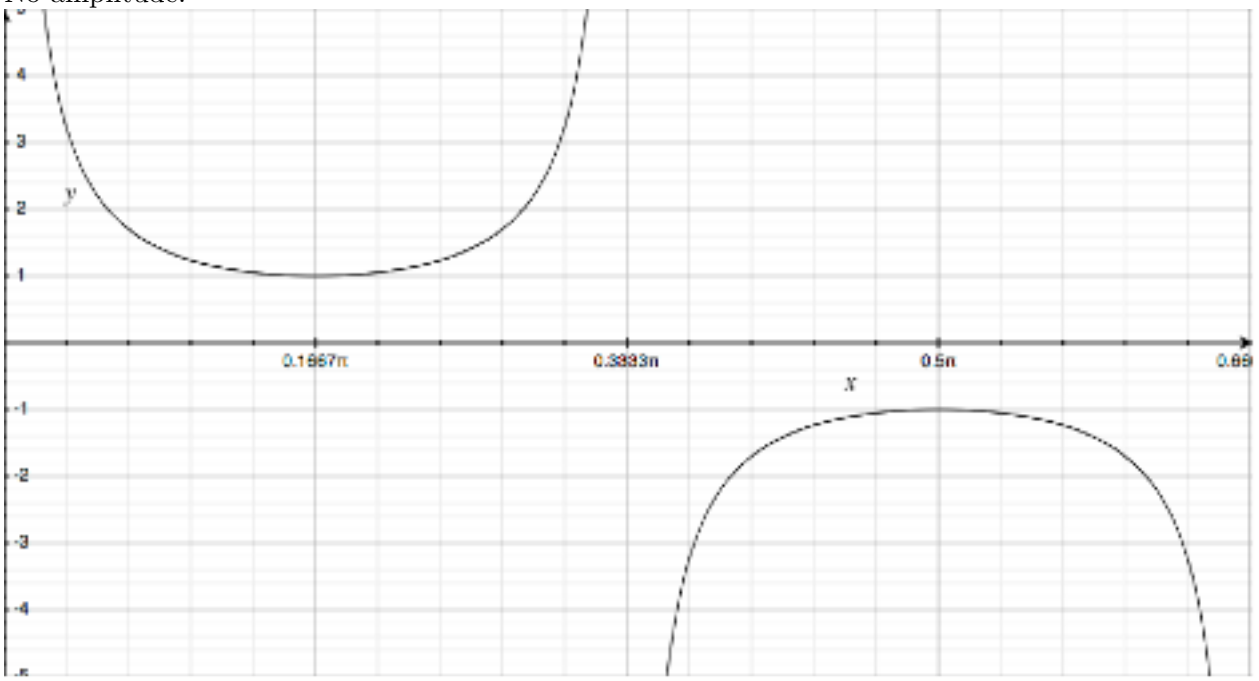
Period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. So it goes through 1 complete cycle every π units on the $x - axis$.

Phase shift is $\frac{-C}{B} = \frac{-\frac{\pi}{2}}{2} = \frac{\pi}{4}$ This is a *positive* $\frac{\pi}{4}$, so we START there. The $x - coordinate$ for the END of 1 cycle is $\frac{\pi}{4} + \pi = \frac{5\pi}{4}$ (add the period to the phase shift).

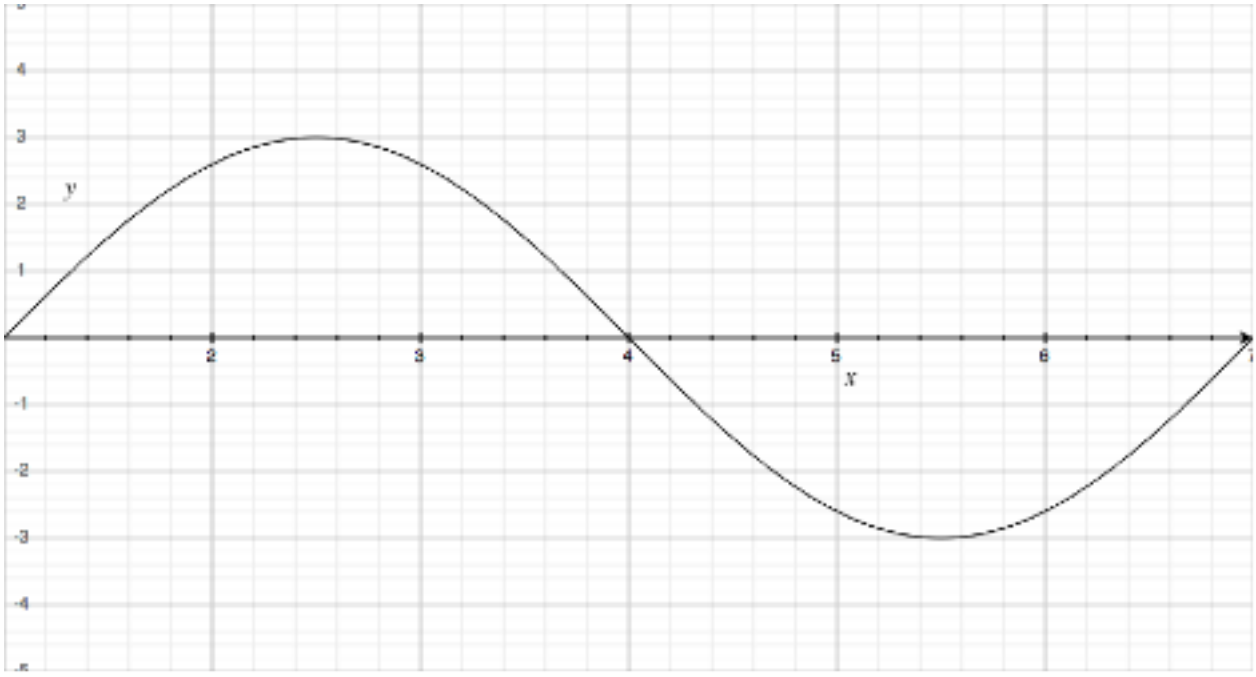
In the process of graphing this 1 cycle, you should divide the interval $[\frac{\pi}{4}, \frac{5\pi}{4}]$ into 4 congruent intervals so that we can compare it to the graph of $y = \cos x$. Each interval should be the length of the period divided by four: i.e., in this case we have $\frac{\pi}{4}$ is the length of each interval.



3. (5 pts) Graph one complete cycle of $y = \csc 3x$ by first graphing the appropriate sine or cosine curve and using reciprocal relationships.
Use the reciprocal function $y = \sin 3x$, which has period $\frac{2\pi}{3}$, and amplitude 1. Taking the period and dividing by four, we obtain the length of each interval: $\frac{\frac{2\pi}{3}}{4} = \frac{\pi}{6}$.
Asymptotes occur whenever $y = \sin 3x$ is 0, so they are at $x = 0$, $x = \frac{\pi}{3}$, $x = \frac{2\pi}{3}$.
No amplitude.



4. (5 pts) Graph one complete cycle of $y = 3\sin(\frac{\pi}{3}x - \frac{\pi}{3})$
Period is $\frac{2\pi}{\frac{\pi}{3}} = 6$
Amplitude is 3
Phase shift is $\frac{\frac{\pi}{3}}{\frac{\pi}{3}} = 1$
Graph is ref START at $x = 1$ (the phase shift), and END when x is $1+6 = 7$ (the phase shift plus the period).
Determine the length of the intervals by dividing the period by four: $\frac{6}{4} = 1.5$.



5. (5 pts each) Each graph below is one complete cycle of the graph of an equation containing a trigonometric function. In each case, find an equation to match the graph.
a. $y = 2 - 4\sin \pi x$
Vertical shift +2
Period is 2, so $\frac{2\pi}{B} = 2$, means $B = \pi$.
No phase shift.

b. $y = -2\cos(3x + \frac{\pi}{2})$

Period is $\frac{\pi}{2} - \frac{-\pi}{6} = \frac{2\pi}{3}$
To find B: $\frac{2\pi}{B} = \frac{2\pi}{3}$, gives us $B = 3$
Phase shift is $-\frac{\pi}{6} = \frac{-C}{B}$, and since we know $B = 3$, we can solve for $C = \frac{\pi}{2}$.
Amplitude is 2
Graph is reflected in the $x - axis$.