Math 104 Quiz #1 Fall 2007

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You must work all of your problems on the quiz. Show ALL of your work and **BOX IN YOUR FINAL ANSWERS.** A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks, class notes, crib sheets, or calculators are not permitted.

1. (6 points) Find all six trigonometric functions of θ if $(-3, \sqrt{7})$ is on the terminal side of θ.

Since $x^2 + y^2 = r^2$, we have r=4 (the radius is always a positive value).
Thus, we have $x = -3, y = \sqrt{7}, r = 4$, so we can obtain the six trigonometric functions:
$\sin\theta = \frac{\sqrt{7}}{4}$
$\cos\theta = \frac{-3}{4}$
$\tan\theta = \frac{-\sqrt{7}}{3}$
$\csc\theta = \frac{4\sqrt{7}}{7}$
$\sec\theta = \frac{-4}{3}$
$\cot\theta = \frac{-3\sqrt{7}}{7}$

2. a. (4 points) Find $\cos\theta$ and $\sin\theta$ if the terminal side of θ lies along the line $y = \sqrt{3}x$ in quadrant III. b. (2 points) How many degrees is θ?

(a) First note that since θ lies in quadrant III, we know that x and y both have negative values. Since the slope of the line is $\frac{\sqrt{3}}{1}$, we know $y = -\sqrt{3}$ and $x = -1$. By the Pythagorean Theorem, $r = 2$, so $\sin\theta = \frac{-\sqrt{3}}{2}$ and $\cos\theta = \frac{-1}{2}$

(b) Note the ratio between the sides of the right triangle that we have created. We have legs of lengths 1, $\sqrt{3}$, and 2–this means that we have a $30 - 60 - 90$ triangle. Adding 60 degrees to 180, we obtain that θ is $60 + 180 = 240$ degrees.

3. (6 points) Find the remaining trigonometric functions of θ if $\sec\theta = 3$ and θ terminates in quadrant IV.

$x = 1$, $r = 3$, so solving for y, we have $y = -2\sqrt{2}$ because θ terminates in quadrant IV.
$\sec\theta = \frac{3}{1}$
$\cos\theta = \frac{1}{3}$
$\csc\theta = \frac{-3\sqrt{2}}{4}$
$\sin\theta = \frac{-2\sqrt{2}}{3}$
$\cot\theta = \frac{-\sqrt{2}}{4}$
$\tan\theta = \frac{-2\sqrt{2}}{1}$

4. (6 points) Show that $\sin\theta(\csc\theta + \cot\theta) = 1 + \cos\theta$ by transforming the left side into the right side.

$\sin\theta(\csc\theta + \cot\theta) = \sin\theta(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}) = \frac{\sin\theta}{\sin\theta} + \frac{\sin\theta\cos\theta}{\sin\theta} = 1 + \cos\theta$

5. (3 points each) Use identity substitutions to simplify:
a. $\csc\theta - \cot\theta\cos\theta$

$\csc\theta - \cot\theta\cos\theta = \frac{1}{\sin\theta} - \frac{\cos\theta\cos\theta}{\sin\theta} = \frac{1-\cos^2\theta}{\sin\theta} = \frac{\sin^2\theta}{\sin\theta} = \sin\theta$

b. $(1 - \sin\theta)(1 + \sin\theta)$

$(1 - \sin\theta)(1 + \sin\theta) = 1 - \sin^2\theta = \cos^2\theta$