

Math 104 Practice Test #2 Fall 2007
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Note that numbers 1-5 are from our Chapter 4 Quiz.

DIRECTIONS: For each graph, label the axes accurately and if applicable, identify the period, amplitude, phase shift, or asymptotes. **Neatness counts!** Make sure your graphs are easy to read.

1. Graph one complete cycle of $y = \tan \frac{1}{2}x$.
2. Graph the equation $y = -4 \cos(2x - \frac{\pi}{2})$, on the interval $-\frac{\pi}{4} \leq x \leq \frac{3\pi}{2}$.
3. Graph one complete cycle of $y = \csc 3x$ by first graphing the appropriate sine or cosine curve and using reciprocal relationships.
4. Graph one complete cycle of $y = 3 \sin(\frac{\pi}{3}x - \frac{\pi}{3})$
5. Each graph below is one complete cycle of the graph of an equation containing a trigonometric function. In each case, find an equation to match the graph.
 - a. page 209 #21 in your text
 - b. page 209 #25 in your text

Do as many other problems like #1-5 as you can. #6-#14 are different variations of these types of problems taken from your textbook:

6. Graph $y = -\cot 2x$ on the interval $0 \leq x \leq \pi$.
7. Graph $y = -2 \sin(-3x)$ on the interval $0 \leq x \leq 2\pi$.
8. Graph $y = 3 \cos \pi x$ on the interval $-2 \leq x \leq 4$.
9. Graph one complete cycle of $y = -1 + \frac{1}{2} \cos 3x$
10. Graph one complete cycle of $y = \sin(\pi x - \frac{\pi}{2})$
11. Graph $y = -\frac{3}{4} \sin(3x - \frac{\pi}{2})$ on the interval $-\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$

12. Graph one complete cycle of $y = 3 \csc(2x + \frac{\pi}{3})$ by first graphing the appropriate sine or cosine curve and then using the reciprocal relationships.

12. Graph one complete cycle of $y = 3 + 3 \cos(\frac{1}{2}x + \frac{\pi}{3})$

13. Graph one complete cycle of $y = \tan(2x + \frac{\pi}{2})$.

14. Each graph below is one complete cycle of the graph of an equation containing a trigonometric function. In each case, find an equation to match the graph. a. pg 208 #13

b. pg 208 #20

c. pg 209 #26

d. pg 209 #27

15. Evaluate each expression without using a calculator, and write your answers in radians:

a. $\arcsin \frac{\sqrt{3}}{2}$

b. $\arccos 0$

c. $\arctan \frac{1}{\sqrt{3}}$

d. $\arcsin -1$

e. $\arccos 1$

f. $\arctan 1$

16. Simplify $5|\sec \theta|$ if $\theta = \arctan \frac{x}{5}$ for some real number x .

17. Evaluate without using a calculator. Show all necessary work to prove that you did not merely plug this into your calculator. If no appropriate work is shown, it will not receive any credit.

a. $\cos(\arctan \frac{3}{4})$

b. $\sin(\arccos \frac{1}{\sqrt{5}})$

c. $\csc(\arctan \frac{3}{4})$

d. $\arctan(\tan \frac{2\pi}{3})$

e. $\arccos(\cos \frac{7\pi}{4})$

f. $\arccos(\sin -\frac{\pi}{6})$

g. $\sin(\arccos -\frac{2}{3})$

18. Prove the following identities:

a. $\sin^2 x(\cot^2 x + 1) = 1$

b. $\frac{\csc x - 1}{\csc x + 1} = \frac{1 - \sin x}{1 + \sin x}$

- c. $\cos^4 A - \sin^4 A = 1 - 2 \sin^2 A$
d. $\frac{\cos x}{1+\sin x} - \frac{1-\sin x}{\cos x} = 0$
e. $\cot x = \frac{\sin 2x}{1-\cos 2x}$
f. $\cos 4x = 8 \cos^4 x - 8 \cos^2 x + 1$
g. $\sin^2 \frac{\theta}{2} = \frac{\csc \theta - \cot \theta}{2 \csc \theta}$
h. $\cos^4 \theta = \frac{1}{4} + \frac{\cos 2\theta}{2} + \frac{\cos^2 2\theta}{4}$

19. Find the exact values for the following:

- a. $\sin 75^\circ$
b. $\cos \frac{7\pi}{12}$
c. $\sin 105^\circ$

20. a. Let $\cos A = -\frac{5}{13}$ with A in QII and $\sin B = \frac{3}{5}$ with B in QI. Find $\sin(A - B)$, $\cos(A - B)$, and $\tan(A - B)$. What quadrant does $A - B$ terminate?

b. If $\tan(A + B) = 2$ and $\tan B = \frac{1}{3}$, find $\tan A$.

21. a. Derive the formula for $\tan(A - B)$ using the formulas for sine and cosine.

b. Find $\tan A$ if $\tan(A - B) = 9$ and $\tan B = \frac{1}{3}$.

22. Let $\sin \theta = \frac{4}{5}$ with θ in the second quadrant and $\sin \alpha = -\frac{12}{13}$ with α in the third quadrant. Compute $\sin(\theta + \alpha)$.

23. Find the exact values of:

- a. $\sin 22.5^\circ$
b. $\tan \frac{\pi}{12}$
c. $\cot 15^\circ$

24. If $x = a \sin \theta$, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ and $a > 0$, express $\sqrt{a^2 - x^2}$ in terms of a trig function of θ .

25. Express as a single trig function and then simplify:

$$\sin \frac{\pi}{12} \cos \frac{5\pi}{12} + \cos \frac{\pi}{12} \sin \frac{5\pi}{12}$$

26. If $x = 4 \sin \theta$, write the expression $\frac{\theta}{2} - \frac{\sin 2\theta}{4}$ in terms of just x .

27. If $\sin A = \frac{4}{5}$ with A in QII, and $\sin B = \frac{3}{5}$ with B in QI, find:

- a. $\sin \frac{A}{2}$

- b. $\cos \frac{A}{2}$
- c. $\cos 2A$
- d. $\sin 2A$
- e. $\sec 2A$
- f. $\csc 2A$
- g. $\cos \frac{B}{2}$
- h. $\sin \frac{B}{2}$
- i. $\sin(A + B)$
- j. $\cos(A + B)$
- k. $\cos(A - B)$
- l. $\sin(A - B)$