

Math 102 Test #1 Fall 2007 w/ Solns

Directions: Show all your work on this paper and box your final answers. No calculators are allowed.

1. (10 pts.) For $f(x) = x^2 - 2x + 3$, find $\frac{f(x+h)-f(x)}{h}$ for $h \neq 0$ and simplify completely.

$$2x - 2 + h.$$

2. Consider the quadratic model $h(t) = -16t^2 + 40t + 50$ for the height h (in feet), of an object t seconds after the object has been projected straight up into the air.

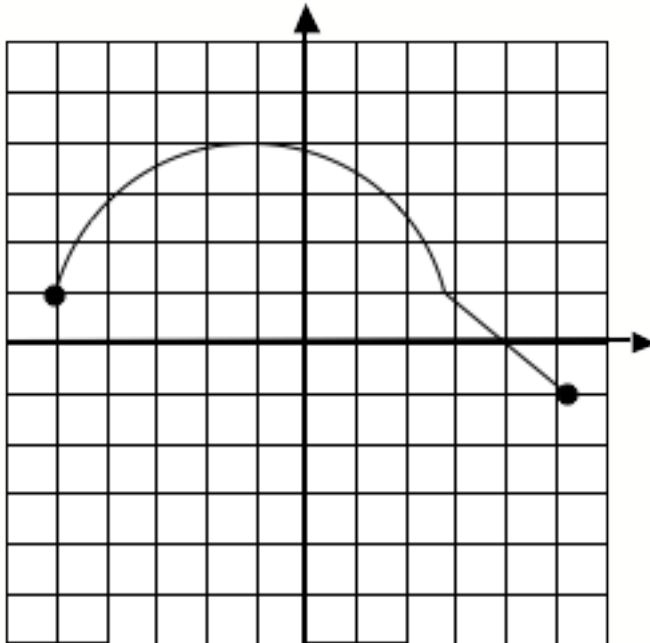
a. (4 pts.) How long does it take the object to attain its maximum height?

$$t = \frac{5}{4} \text{seconds.}$$

b. (4 pts.) What is its maximum height?

$$h\left(\frac{5}{4}\right) = 75 \text{feet}$$

3. Use the graph of the function f below to answer the following:



a. (2 pts.) What is $f(-1)$?

$$f(-1) = 4$$

b. (2 pts.) On what interval is $f(x) \geq 0$?

$$[-5, 4]$$

c. (2 pts.) What is a local maximum value of f ?

$$4$$

d. (2 pts.) On what interval is f increasing?

$$[-5, -1]$$

e. (2 pts.) What is the range of f ?

$$[-1, 4]$$

4. Let $g(x) = \sqrt{-x} + 3$.

a. (4 pts.) Write a sentence describing how the graph of $g(x)$ can be obtained from the graph of $f(x)$ using transformations.

Reflect $f(x)$ in the y -axis, and then shift vertically up 3 units.

b. (6 pts.) Sketch the graph of $g(x)$, and include at least 3 labeled points on your graph.

Notice that the domain of $g(x)$ is $(-\infty, 0]$. I used the following points on my graph: $(-4, 5)$, $(-1, 4)$, $(0, 3)$.

5. Let

$$f(x) = \begin{cases} x^3 & \text{if } x < 0; \\ 1 & \text{if } 0 \leq x \leq 2; \\ x + 1 & \text{if } x > 2. \end{cases}$$

a. (2 pts.) Find $f(-2)$.

$$f(-2) = -8$$

b. (2 pts.) Find $f(\frac{1}{2})$.

$$f(\frac{1}{2}) = 1$$

c. (8 pts.) Sketch the graph of f , and include at least 3 labeled points on your graph.

see me for the graph.

6. Let $P = (x, y)$ be a point on the graph of $y = \sqrt{x}$.
- a. (6 pts.) Express the distance d from P to the point $(1, 0)$ as a function of x . (Hint: Recall that the distance formula is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$).
- $d(x) = \sqrt{x^2 - x + 1}$
- b. (2 pts.) What is d if $x = 2$?
- $d(2) = \sqrt{3}$

7. The monthly cost C , in dollars for international calls on a certain cellular phone plan is given by the function $C(x) = \frac{1}{4}x + 5$, where x is the number of minutes used.

- a. (5 pts.) What is the cost if you talk on the phone for 60 minutes?
- $C(60) = 20$ dollars.
- b. (5 pts.) Suppose you budget yourself for \$57 per month for the phone. What is the maximum number of minutes you can talk?
- 208 minutes

8. Let $f(x) = -x^2 + 4x + 5$.

- a. (4 pts.) Find the vertex of f .
- vertex = $(\frac{-b}{2a}, f(\frac{-b}{2a})) = (2, 9)$
- b. (2 pt.) Which does f have, a maximum or minimum value?
- Maximum because the parabola opens downward.
- c. (2 pts.) Find the equation of the axis of symmetry of f .
- $x = 2$
- d. (4 pts.) Sketch the graph of f . Label the x and y intercepts, and the vertex.
- x -intercepts are $(5, 0)$ and $(-1, 0)$. y -intercept is $(0, 5)$.

9. A rectangular field is to be enclosed with 280 yards of fencing. One side of the field faces a river and does not need fencing.

- a. (6 pts.) Express the area of the field A as a function of its width x .
- $A(x) = -2x^2 + 280x$
- b. (4 pts.) For what value of x will the area be the greatest?
- $x = 70$ yards.

10. (10 pts.) Find the average rate of change of the function $f(x) = \sqrt[3]{1-x}$ on the interval $[-7, 9]$.

$\frac{-1}{4}$