MSE 362

Reliability Evaluation of Engineering Systems

Concepts and Techniques

Second Edition

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4 Network modelling and evaluation of simple systems

4.1 Network modelling concepts

The previous chapters have considered the application of basic probability techniques to combinational types of reliability assessment. In many types of problems these techniques may be all that is required to assess the adequacy of the system. However, in practice, a system is frequently represented as a network in which the system components are connected together either in series, parallel, meshed or a combination of these. This chapter considers series and parallel network representations (more complicated meshed networks are considered in the next chapter).

It is vital that the relationship between the system and its network model be thoroughly understood before considering the analytical techniques that can be used to evaluate the reliability of these networks.

It must be appreciated that the actual system and the reliability network used to model the system may not necessarily have the same topological structure. This consideration involves the key point discussed previously that, before a reliability assessment of a system can be made, the analyst must be fully conversant with the requirements of the system and be able to phrase these requirements in a form which can be quantitatively assessed.

Definitions of series systems and parallel systems as represented in a reliability network are considered first.

(a) *Series systems*

The components in a set are said to be in series from a reliability point of view if they must *all* work for system success or only *one* needs to fail for system failure.

(b) *Parallel systems*

The components in a set are said to be in parallel from a reliability point of view if only *one* needs to be working for system success or *all* must fail for system failure.

These definitions provide a link between the present discussion and that of Chapter 3 on the use of the binomial distribution. A series system therefore represents a non-redundant system and a parallel system represents a fully redundant system. Reconsider Example 3.7 in which four operating conditions were applied to a four component system. The condition of 'all four components must be working for system success' could be represented by a network in which all four components are connected in series. Similarly, the condition of 'only one component need be working for system success' could be represented by a network in which all four components are connected in parallel. At this stage it appears that the series and parallel network models are simply additional methods for representing non-redundant and fully redundant systems whereas the previous techniques were able to solve such systems and more. Series and parallel network considerations are used extensively in system appreciation, representation and reduction. They are used in a wide range of applications including extremely simple models and complicated systems with complex operational logic. Modelling of these systems using the techniques described in Chapter 3 becomes difficult if not impossible.

The main points in reconsidering Example 3.7 are: (a) there is frequently more than one way of solving the same problem, (b) there are frequently links between one technique and another, and (c) most importantly, a physical system having a defined topological structure may have a considerably different reliability network topology which itself may change when the requirements of the physical system change although the topology of the physical system remains the same.

It remains with the individual analyst to consider the actual requirements of a system and to construct a reliability network from these requirements. The following analytical techniques can then be applied to this reliability network. A reliability network is often referred to as a reliability block diagram.

4.2 Series systems

Consider a system consisting of two independent components A and B connected in series, from a reliability point of view, as shown in Figure 4.1. This arrangement implies that both components must work to ensure system success. Let R_A, R_B = probability of successful operation of components A and B respectively, and Q_A, Q_B = probability of failure of

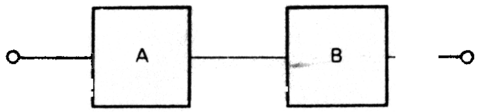

Fig. 4.1 Two component series system

components A and B respectively. Since success and failure are mutually exclusive and complementary,

$$R_A + Q_A = 1 \quad \text{and} \quad R_B + Q_B = 1$$

The requirement for system success is that 'both A and B' must be working. Equation 2.9 can be used to give the probability of system success or reliability as

$$R_S = R_A \cdot R_B \tag{4.1}$$

If there are now n components in series, Equation 4.1 can be generalized to give

$$R_S = \prod_{i=1}^{n} R_i \tag{4.2}$$

This equation frequently is referred to as the product rule of reliability since it establishes that the reliability of a series system is the product of the individual component reliabilities.

In some applications it may be considered advantageous to evaluate the unreliability or probability of system failure rather than evaluating the reliability or probability of system success. System success and system failure are complementary events and therefore for the two component system the unreliability is

$$Q_S = 1 - R_A R_B \tag{4.3}$$

$$= 1 - (1 - Q_A)(1 - Q_B)$$

$$= Q_A + Q_B - Q_A \cdot Q_B \tag{4.4}$$

or for an n component system,

$$Q_S = 1 - \prod_{i=1}^{n} R_i \tag{4.5}$$

Equations 4.3 and 4.5 could have been derived directly from Equation 2.12 since the requirement for system failure is that 'A or B or both' must fail.

Now consider the application of these techniques to some specific problems.

Example 4.1

A system consists of 10 identical components, all of which must work for system success. What is the system reliability if each component has a reliability of 0.95?

From Equation 4.2,

$$R_S = 0.95^{10} = 0.5987$$

This result, although easily derived, establishes an important concept concerning the reliability of series systems. Because of the product rule and the fact that each component has a probability of success that is less than unity, the system reliability is less than the reliability of any one component. It also decreases as the number of components in series increases and as the component reliability decreases. This is illustrated in Figure 4.2 which shows the reliability of series systems containing identical components as a function of the number of series components and the component reliability. It is evident from these results that the system reliability decreases very rapidly as the number of series components increases, particularly for those systems in which the components do not have a very high individual reliability.

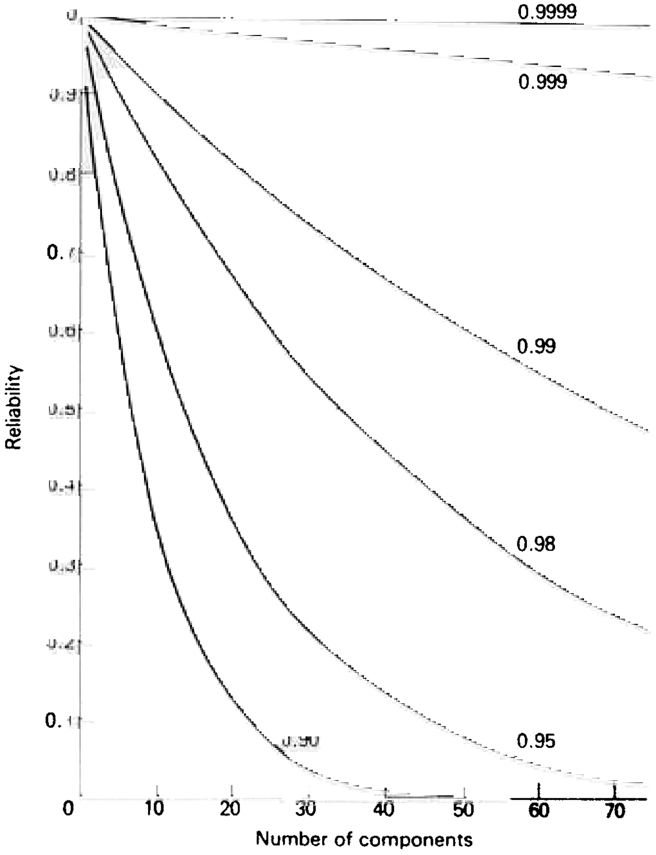

Fig. 4.2 Effects of increasing the number of series components. The numbers represent the reliability of each single component

Example 4.2

A two component series system contains identical components each having a reliability of 0.99. Evaluate the unreliability of the system.

From Equation 4.5, $Q_S = 1 - 0.99^2 = 0.0199$

From Equation 4.4, $Q_S = 0.01 + 0.01 - (0.01 \times 0.01) = 0.0199$

These two results are identical as would be expected. In some system analyses however, Equation 4.4 is used in an approximate form, i.e. the product term which is subtracted from the summation terms is neglected. If this was done in the present example:

$Q_S \simeq 0.01 + 0.01 = 0.02$

which gives an error of 0.5%.

This approximation must be used with extreme care and applies only if the number of components is small and the reliability of each component is very high. The approximation is not used again in this chapter but is re-introduced in the next chapter in connection with approximate techniques used for analysing complex systems. Its advantage is that the reliability of a series system can be evaluated from the product of component reliabilities and the unreliability of the system from a summation of component unreliabilities.

In the design of a complex system or plant, a design parameter that may be specified is the overall system reliability. From this overall value, the required reliability of the system's components is then evaluated. This is the inverse procedure of that used in the previous examples and is illustrated in the following example.

Example 4.3

A system design requires 200 identical components in series. If the overall reliability must not be less than 0.99, what is the minimum reliability of each component?

From Equation 4.2

$0.99 = R^{200}$

i.e. $R = 0.99^{1/200} = 0.99995$

4.3 Parallel systems

Consider a system consisting of two independent components A and B, connected in parallel, from a reliability point of view as shown in Figure 4.3.

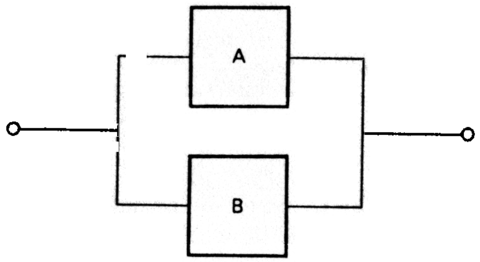

Fig. 4.3 The two component parallel system

In this case the system requirement is that only one component need be working for system success. The system reliability can be obtained as the complement of the system unreliability or by using Equation 2.12 since 'either A or B or both' constitutes success to give

$$R_P = 1 - Q_A \cdot Q_B \tag{4.6}$$

$$= R_A + R_B - R_A \cdot R_B \tag{4.7}$$

or for an n component system:

$$R_P = 1 - \prod_{i=1}^{n} Q_i \tag{4.8}$$

Also

$$Q_P = Q_A \cdot Q_B \tag{4.9}$$

and for an n component system:

$$Q_P = \prod_{i=1}^{n} Q_i \tag{4.10}$$

It follows that the equations for a parallel system are of the same form as those of a series system but with R and Q interchanged. In the case of parallel systems, Equation 4.10 leads to the concept of the product rule of unreliabilities. However, unlike the case of series systems in which, under certain circumstances, Equation 4.4 can be reduced to a simple summation, Equation 4.7 cannot be simplified in this way since the product $(R_A \cdot R_B)$ is, hopefully, always reasonably comparable with the values of R_A and R_B.

In the case of series systems, the system reliability decreased in the number of series components was increased following Equation 4.2. In the case of parallel systems, however, it is the unreliability that decreases as the number of parallel components is increased following Equation 4.10 and hence the reliability increases with the number of components. Increasing the number of parallel components increases the initial cost, weight and volume of the system and increases the required maintenance. Therefore, it must be examined very carefully.

In order to illustrate the application of the equations for parallel systems consider the following examples.

Example 4.4

A system consists of four components in parallel having reliabilities of 0.99, 0.95, 0.98 and 0.97. What is the reliability and unreliability of the system?

From Equation 4.10 $Q_P = (1-0.99)(1-0.95)(1-0.98)(1-0.97)$

$$= 3 \times 10^{-7}$$

and from Equation 4.8 $R_P = 0.9999997$

This example also demonstrates the difficulty of physically appreciating the quality of a system in terms of the reliability value R since for many practical systems this numerical value is often a series of 9s followed by another digit or more. It is often more reasonable to state the unreliability as this eliminates the string of 9s and provides a value that is more easily interpreted.

Example 4.5

A system component has a reliability of 0.8. Evaluate the effect on the overall system reliability of increasing the number of these components connected in parallel.

Using Equation 4.10, the value of system reliability is shown in Table 4.1 for systems having 1 to 6 components in parallel. Also shown in Table 4.1 is the increase in reliability obtained by adding each additional component. This is known as incremental reliability. The percentage comparative reliability defined as the change in reliability over that of a

Table 4.1 Reliability results for Example 4.5

Number of components	*System reliability*	*Incremental reliability*	*Percentage comparative reliability*
	0.800000	—	
2	0.960000	0.160000	20.00
3	0.992000	0.032000	24.00
4	0.998400	0.006400	24.80
5	0.999680	0.001280	24.96
6	0.999936	0.000256	24.99

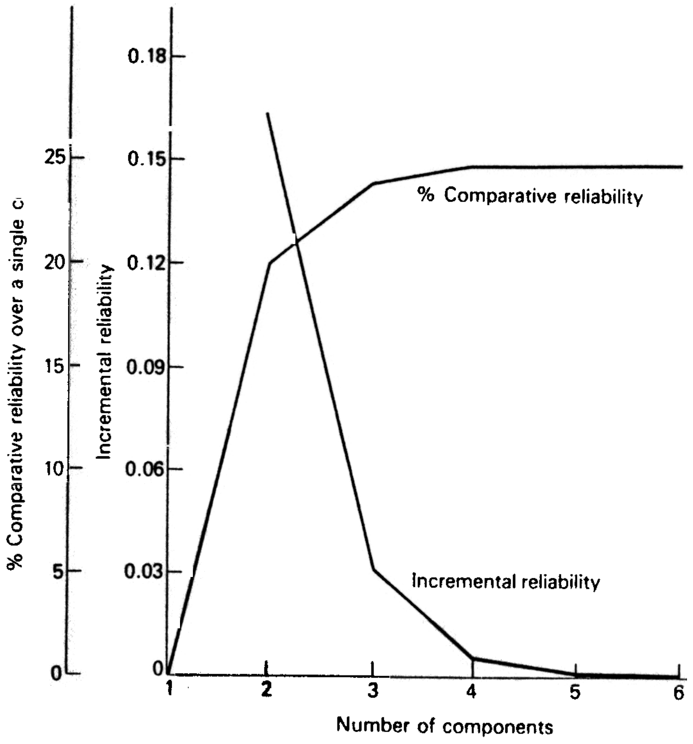

Fig. 4.4 Incremental reliability benefits

single component expressed as a percentage based on the single component reliability is also shown in Table 4.1. The results for incremental reliability and comparative reliability are also shown in Figure 4.4.

From Figure 4.4 it is evident that the addition of the first redundant component to the one-component system provides the largest benefit to the system, the amount of improvement diminishing as further additions are made.

Since the abscissa axis of Figure 4.4 is related to the cost of the system, an incremental worth–cost analysis can be performed using diagrams of incremental reliability such as that shown in Figure 4.4.

Example 4.6

A system is to be designed with an overall reliability of 0.999 using components having individual reliabilities of 0.7. What is the minimum number of components that must be connected in parallel?

From Equation 4.10

$$(1-0.999)=(1-0.7)^n$$

$$0.001=0.3^n$$

$$n=5.74$$

since the number of components must be an integer, the minimum number of components is 6.

It should be noted that increasing the number of parallel elements may actually decrease the reliability of the system if a component failure mode exists which in itself causes a system failure. An example of this is described in Section 5.9.

4.4 Series–parallel systems

The series and parallel systems discussed in the two previous sections form the basis for analysing more complicated configurations. The general principle used is to reduce sequentially the complicated configuration by combining appropriate series and parallel branches of the reliability model until a single equivalent element remains. This equivalent element then represents the reliability (or unreliability) of the original configuration. The following examples illustrate this technique which is generally known as a (network) reduction technique.

Example 4.7

Derive a general expression for the reliability of the model shown in Figure 4.5 and hence evaluate the system reliability if all components have a reliability of 0.9.

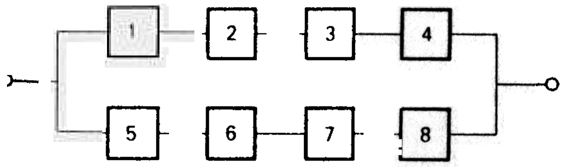

Fig. 4.5 Reliability diagram of Example 4.7

This model could represent, for example, a duplicated control circuit associated with the automatic pilot of an aeroplane. The reduction process is sequential and proceeds as follows.

Combine in series components 1–4 to form an equivalent component 9, combine in series components 5–8 to form an equivalent component 10 and then combine in parallel equivalent components 9 and 10 to form an equivalent component 11 that represents the complete system. This logical step process is illustrated in Figure 4.6.

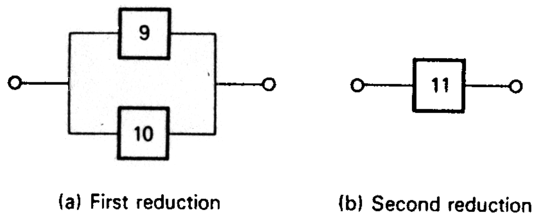

Fig. 4.6 Reduction of Example 4.7.

If $R_1, R_2, \ldots, R_8$ are the reliabilities of components $1, 2, \ldots, 8$ respectively then

$$R_9 = R_1R_2R_3R_4$$
$$R_{10} = R_5R_6R_7R_8$$
$$R_{11} = 1-(1-R_9)(1-R_{10})$$
$$= R_9 + R_{10} - R_9R_{10}$$
$$= R_1R_2R_3R_4 + R_5R_6R_7R_8 - R_1R_2R_3R_4R_5R_6R_7R_8$$

In deriving expressions of this type, it is possible to produce a number of apparently different equations when the final expression is written in terms of both Rs and Qs. These apparently different versions could all be correct and should reduce to the same one if manipulated and expressed in terms of either R or Q.

Using the data of Example 4.7, then

$$R_{11} = 0.9^4 + 0.9^4 - 0.9^8 = 0.8817$$

Example 4.8

Derive a general expression for the unreliability of the model shown in Figure 4.7 and hence evaluate the unreliability of the system if all components have a reliability of 0.8.

The logical steps for this example are: combine components 3 and 4 to form equivalent component 6, combine components 1 and 2 with equivalent component 6 to give equivalent component 7 and finally combine component 5 with equivalent component 7 to give equivalent component 8 that represents the system reliability. These reduction steps are shown in Figure 4.8.

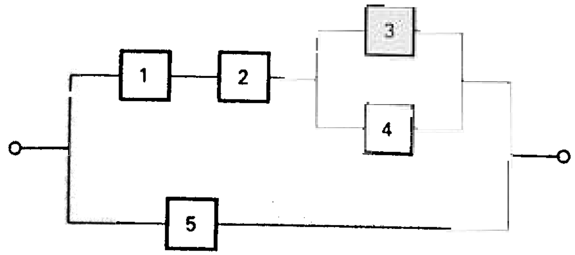

Fig. 4.7 Reliability diagram for Example 4.8

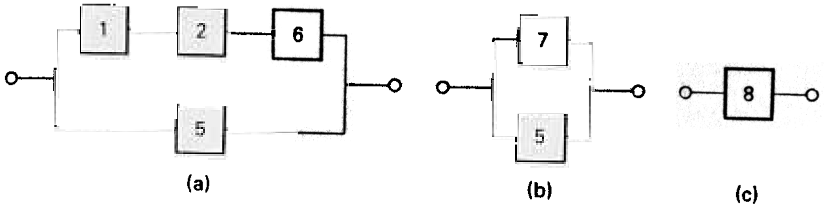

Fig. 4.8 Reduction of Example 4.8. (a) First reduction. (b) Second reduction. (c) Third reduction

If $R_1, \ldots, R_5$ and $Q_1, \ldots, Q_5$ are the reliabilities and unreliabilities of components $1, \ldots, 5$ respectively, then

$$Q_6 = Q_3Q_4$$
$$Q_7 = 1-(1-Q_1)(1-Q_2)(1-Q_6)$$
$$= Q_1 + Q_2 + Q_6 - Q_1Q_2 - Q_2Q_6 - Q_6Q_1 + Q_1Q_2Q_6$$
$$Q_8 = Q_5Q_7$$
$$= Q_5(Q_1 + Q_2 + Q_3Q_4 - Q_1Q_2 - Q_2Q_3Q_4 - Q_3Q_4Q_1 + Q_1Q_2Q_3Q_4)$$

For the data given, $R_i = 0.8$ thus $Q_i = 0.2$ and $Q_8 = 0.07712$.

An equivalent expression to the above could have been deduced in terms of R_i.

$$R_6 = R_3 + R_4 - R_3R_4$$
$$R_7 = R_1R_2R_6$$
$$R_8 = R_5 + R_7 - R_5R_7$$
$$= R_5 + R_1R_2(R_3 + R_4 - R_3R_4) - R_5R_1R_2(R_3 + R_4 - R_3R_4)$$

which, for $R_i = 0.8$, gives:

$$R_8 = 0.92288 \quad \text{or} \quad Q_8 = 1 - 0.92288 = 0.07712$$

4.5 Partially redundant systems

The previous sections have been concerned only with series systems (non-redundant) and parallel systems (fully redundant). In many systems, these two extreme situations are not always applicable as there may be some parts of the system that are partially redundant. The concepts of partial redundancy were presented in Chapter 3 in the discussion of the binomial distribution. The techniques described in this chapter for series/parallel systems cannot be used directly for cases involving partial redundancy. The principles that have been described together with the inclusion of binomial distribution concepts can, however, enable any series–parallel system containing regions of partial redundancy to be evaluated.

In order to illustrate this intermingling of the two techniques, consider the following example.

Example 4.9

Derive a general expression for the unreliability of the system whose reliability model is shown in Figure 4.9. Consider the case in which all parallel branches of this system are fully redundant with the exception of that consisting of components 4, 5 and 6 for which any 2 of the branches are required for system success.

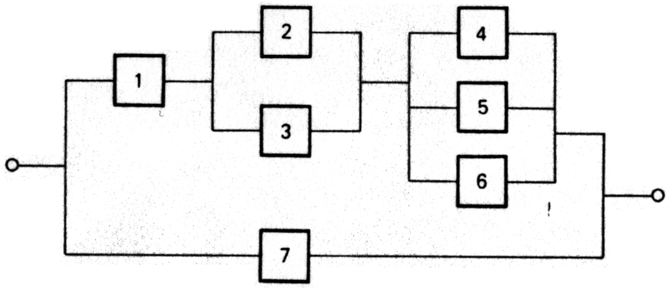

Fig. 4.9 Reliability diagram of Example 4.9

The principle of network reduction applies equally well to this problem, i.e., components 2 and 3 are combined to give equivalent component 8; components 4, 5 and 6 are combined to give equivalent component 9, component 1 and equivalent components 8 and 9 are combined to give equivalent component 10 and finally equivalent component 10 is combined with component 7 to give the system equivalent component 11. These steps are shown in Figure 4.10.

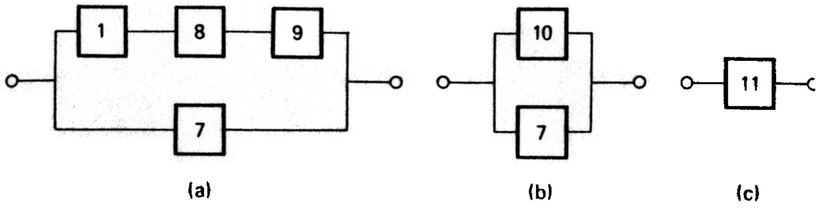

Fig. 4.10 Reduction of Example 4.9. (a) First reduction. (b) Second reduction. (c) Third reduction

The only essential difference between this example and those considered previously in this chapter is that the reliability of equivalent component 9 cannot be evaluated using the equations of Section 4.3 but, instead, must be evaluated from the binomial distribution concepts described in Chapter 3. The binomial distribution can be applied directly if components 4, 5 and 6 are identical. A fundamentally similar approach is used for non-identical components.

If $R_1, \ldots, R_7$ and $Q_1, \ldots, Q_7$ are the reliabilities and unreliabilities of components $1, \ldots, 7$, then

$$Q_8 = Q_2 Q_3$$

$$R_{10} = R_1 R_8 R_9$$

$$\begin{aligned} Q_{11} &= Q_{10} Q_7 \\ &= Q_7(1 - R_1 R_8 R_9) \\ &= Q_7(1 - R_1(1 - Q_2 Q_3) R_9) \\ &= Q_7(1 - R_1 R_9 + R_1 R_9 Q_2 Q_3) \end{aligned}$$

R_9 is evaluated by applying the binomial distribution to components 4, 5 and 6.

If $R_4 = R_5 = R_6 = R$ and $Q_4 = Q_5 = Q_6 = Q$, then

$$R_9 = R^3 + 3R^2 Q$$

and $Q_9 = 3RQ^2 + Q^3$

If $R_4 \neq R_5 \neq R_6$ and $Q_4 \neq Q_5 \neq Q_6$, then

$$R_9 = R_4 R_5 R_6 + R_4 R_5 Q_6 + R_5 R_6 Q_4 + R_6 R_4 Q_5$$

and $Q_9 = R_4 Q_5 Q_6 + R_5 Q_6 Q_4 + R_6 Q_4 Q_5 + Q_4 Q_5 Q_6$

In the special case when all components have a reliability of 0.8

$$R_9 = 0.8960, \qquad Q_9 = 0.1040$$

and $Q_{11} = 0.06237$.

4.6 Standby redundant systems

4.6.1 Redundancy concepts

In Section 4.3 it was assumed that a redundant system consisted of two or more branches connected in parallel and that both branches were operating simultaneously.

In some system problems, however, one or more branches of the redundant components may not be continuously operating but remain, in normal operating circumstances, in a standby mode, i.e., they are only switched into an operating mode when a normally operating component fails. The essential difference between these two types of redundancy is illustrated in Figure 4.11.

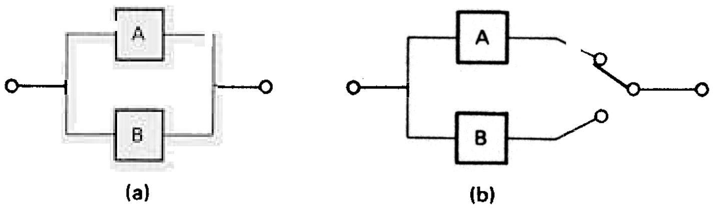

Fig. 4.11 Redundancy modes. (a) Parallel redundancy. (b) Standby redundancy

It is not the purpose of this book to explain which of the two redundancy modes should be used in any specific engineering application but to recognize that both can exist in practice and to explain how each can be analysed. It is however worth mentioning some of the factors involved in deciding which is most appropriate.

In some applications it is physically not possible for both branches to be operating. This could occur for instance when both A and B in Figure 4.11 are used to control some other device. If for some reason A and B produced different outputs, the device would receive opposing instructions. This can be overcome by including a logic gate between the parallel branches and the device so that both A and B may operate but the device receives only one set of instructions and the other is blocked by the gate. In other applications it may be preferable for a component to remain idle unless required to operate following the malfunction of another component because its probability of failure may be insignificant when not operating compared to its probability of failure when in an active and operating mode. This is frequently the case for mechanical devices such as motors and pumps. In such cases, standby redundancy is more appropriate. There are instances in which the failure probability of a component or system is less, when continuously operated, compared with that when it

is frequently cycled between an idle and inactive mode and an active mode. This can occur in the case of computer systems and frequently when two or more are used in a redundant process or control application, they are allowed to share the operating duties and each have the ability to pick up the duties of another if the latter should fail during operation. Both modes of redundancy exist in practice and the reader will no doubt be able to compile a list of alternative applications of each.

In the case of standby redundancy, the additional features that exist are the cyclic duty of the redundant component(s) and the necessity of switching from one branch to another.

4.6.2 Perfect switching

Consider the case of a perfect switch, i.e., it does not fail during operation and does not fail in switching from the normal operating position to the standby position. A typical standby redundant system can be as shown in Figure 4.11b.

If it is assumed that B does not fail when in the standby position, then it can only fail given that A has already failed, i.e. B is operating.

Therefore, the failure of this system is given by failure of A and failure of B, given A has failed.

Using the symbolism of Chapter 2, the probability of system failure is

$$Q = Q(\mathrm{A}) \cdot Q(\mathrm{B} \mid \bar{\mathrm{A}})$$

which, if it is assumed that A and B are independent, reduces to:

$$Q = Q_{\mathrm{A}} \cdot Q_{\mathrm{B}} \qquad (4.11)$$

Equation 4.11 appears to be identical to Equation 4.9 and gives the impression that the probability of failure of a standby redundant system is identical to that of a parallel redundant system. This is not true however since the numerical values used in Equations 4.11 and 4.9 are different. Since B is used only for short periods it is not likely that its failure probability will be the same as if it is used continuously. This leads to the necessity of considering time dependent probabilities, whereas up to this point only time independent probabilities have been considered, i.e., it has been assumed that the probabilities do not change with the time for which the component is exposed to failure. Time dependent probabilities are considered in Chapter 7 and the problem of standby redundancy is reconsidered then.

4.6.3 Imperfect switching

Consider the situation in which the switch has a probability of failing to change over from the branch containing component A to that containing component B when A fails. Let the probability of a successful changeover be P_S and the probability of an unsuccessful change over be $\bar{P}_S(=1-P_S)$.

The problem can now be solved using the conditional probability approach discussed in Chapter 2.

$$\begin{aligned}P(\text{system failure}) &= P(\text{system failure } \textit{given} \text{ successful changeover})\\ &\quad \times P(\text{successful changeover})\\ &\quad + P(\text{system failure } \textit{given} \text{ unsuccessful changeover})\\ &\quad \times P(\text{unsuccessful changeover})\end{aligned}$$

$$\begin{aligned}\text{therefore } Q &= Q_A \cdot Q_B \cdot P_S + Q_A \cdot \bar{P}_S\\ &= Q_A Q_B P_S + Q_A(1-P_S)\\ &= Q_A Q_B P_S + Q_A - Q_A P_S\\ &= Q_A - Q_A P_S(1-Q_B) \qquad (4.12)\end{aligned}$$

The value of Q_B in this equation is affected by the time dependent problem of B being operated for short periods as discussed in Section 4.6.2.

Now consider the situation encountered if the switch can fail in its initial operating position as well as failing to change over when required. Since the failure of the switch in its operating position is likely to be identical whether it is connected to A or to B, it can be considered as a component in series with the parallel branch formed by A and B. This leads to the network model shown in Figure 4.12 in which the switch appears as two components; the first represents its switching mode with a probability of successful changeover of P_S and the second represents its normal operating mode with a reliability of R_S and an unreliability of Q_S.

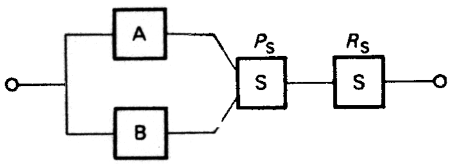

Fig. 4.12 Standby redundancy with imperfect switch

As the second component representation of the switch is in series with the previously considered standby redundant branches, the probability of system failure (or success) can be found by combining its effect with Equation 4.12 to give

$$Q = [Q_A - Q_A P_S(1-Q_B)] + Q_S - [Q_A - Q_A P_S(1-Q_B)]Q_S \qquad (4.13)$$

$$\text{or} \quad R = R_S(1-(Q_A - Q_A P_S(1-Q_B)) \qquad (4.14)$$

4.6.4 Standby redundancy calculations

Example 4.10

Consider Figure 4.11b. Evaluate the reliability of this system if A has a reliability of 0.9, B has a reliability given A has failed of 0.96 and,
(a) the switch is perfect,
(b) the switch has a probability of failing to changeover of 0.08, and
(c) as (b) but the switch has an operating reliability of 0.98.

(a) from Equation 4.11, $R = 1-0.1\times0.04 = 0.996$

(b) from Equation 4.12, $R = 1-(0.1-0.1\times0.92(1-0.04))$
$= 0.988$

(c) from Equation 4.14, $R = 0.98\times0.988 = 0.969$

Example 4.11

Consider the system model shown in Figure 4.13 and assume that A, B and S have the reliability indices given in Example 4.10 and part (c). If components C and D have reliabilities of 0.99 and 0.8 respectively, evaluate the reliability of the system.

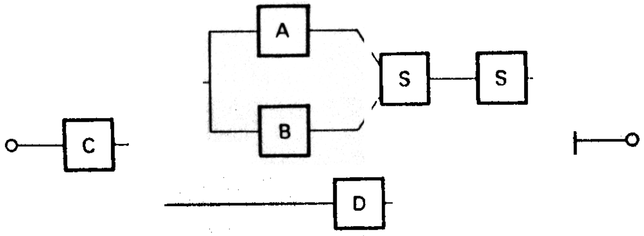

Fig. 4.13 Reliability diagram of Example 4.11

The reliability of this system can be evaluated using the network reduction technique described for series–parallel systems by first evaluating the equivalent component representing A, B and S, combining this with D and finally combining this result with C.

The reliability of the branch containing A, B and S is given in Example 4.10 as

$$R = 0.969$$

The reliability of the system is therefore given by

$$\begin{aligned}R &= R_C(1-Q_D(1-0.969))\\ &= 0.99(1-0.2(1-0.969)) = 0.984\end{aligned}$$

4.7 Conclusions

This chapter has illustrated network modelling of systems and the reliability evaluation of these networks. The discussion has focused on series, parallel redundant and standby redundant systems as well as combinations of these. More complex arrangements require additional techniques which are described in the next chapter.

In network modelling of systems, the reliability network is frequently not identical to the physical system or network. The analyst must translate the physical system into a reliability network using the system operational logic and a sound understanding of the physical behaviour and requirements of the system.

The examples used in this chapter have shown how an increasing number of series components decreases the system reliability whilst an increasing number of parallel and standby redundant components increases the system reliability. The choice of parallel, or standby redundant systems, must be made by the system designer using engineering knowledge of the performance of the components and devices. The merits of each method are affected by the physical requirements of the system and by the difference in reliability of the component or device in the respective modes of redundancy.

Problems

The system shown in Figure 4.14 is made up of ten components. Components 3, 4 and 5 are not identical and at least one component of this group must be available for system success. Components 8, 9 and 10 are identical and for this particular group it is necessary that two out of the three components functions

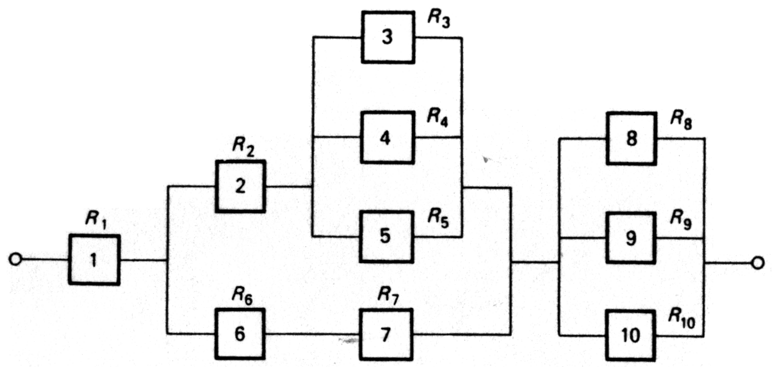

Fig. 4.14

satisfactorily for system success. Write an expression for the system reliability in terms of the R values given. Also evaluate the system reliability if the reliability of each component = 0.8.

2 A system consists of four components in parallel. System success requires that at least three of these components must function. What is the probability of system success if the component reliability is 0.9? What is the system reliability if five components are placed in parallel to perform the same function?

3 A system contains two subsystems in series. System 1 has four possible operating levels, and System 2 has three possible operating levels as shown in the following table.

System 1		*System 2*	
Output	*Probability*	*Output*	*Probability*
100%	0.8	100%	0.7
75%	0.1	50%	0.1
25%	0.05	0%	0.2
0%	0.05		

Develop an operating level probability table for the system.

4 A series system has 10 identical components. If the overall system reliability must be at least 0.99, what is the minimum reliability required of each component?

5 A series system has identical components each having a reliability of 0.998. What is the maximum number of components that can be allowed if the minimum system reliability is to be 0.90?

6 A parallel system has 10 identical components. If the overall system reliability must be at least 0.99, how poor can these components be?

7 A parallel system has identical components having a reliability of 0.5. What is the minimum number of components if the system reliability must be at least 0.99?

8 Write an expression for the reliability of the system shown in Figure 4.15.

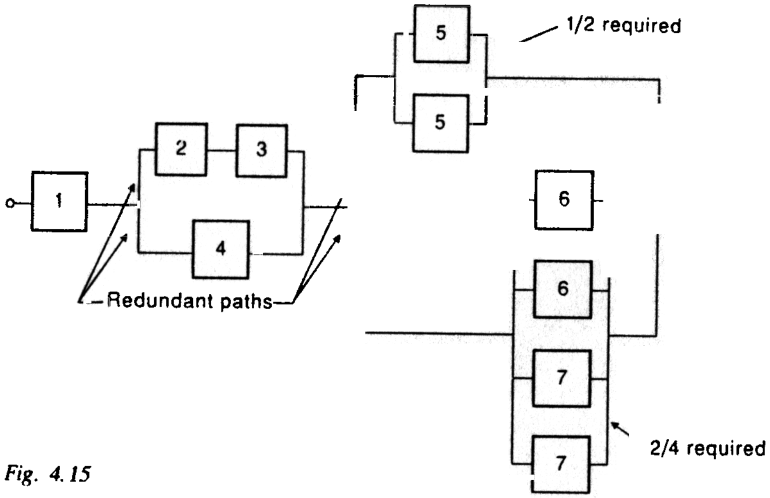

Fig. 4.15

(11)

What is the system reliability if

$$R_1 = R_3 = R_5 = R_7 = 0.85 \text{ and } R_2 = R_4 = R_6 = 0.95?$$

9 Consider the reliability block diagram shown in Figure 4.16. System success requires at least one path of subsystem 1 and at least two paths of subsystem 2 to be working. Evaluate the reliability of the system if the reliability of components 1–6 is 0.9, the reliability of component 7 is 0.99 and the reliability of components 8–10 is 0.85. How many of these systems must be connected in parallel to achieve a minimum system reliability of 0.999?

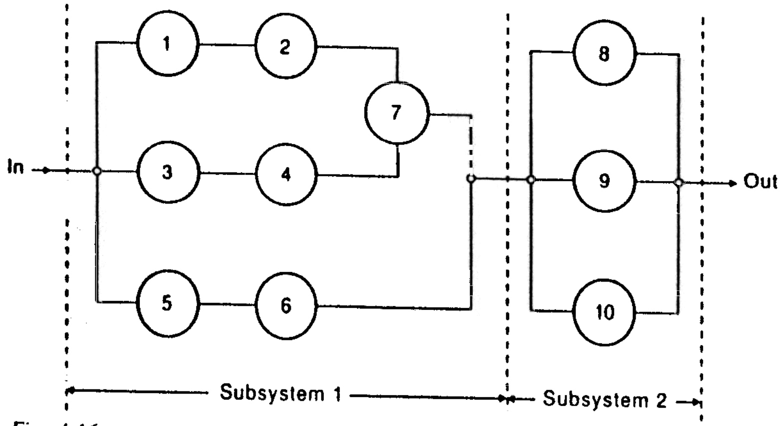

Fig. 4.16

Exponential distribution:

probability density function:

$$f(x) = \lambda e^{-\lambda x}, \text{ for } 0 \le x \le \infty$$

$$E(x) = \frac{1}{\lambda} \; ; \; V(x) = \frac{1}{\lambda^2}$$