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Math 320. Professor Ábrego

Quiz 1

1. Let x and y be integers. Determine the truth value of each statement below.

- (a) xy is odd if and only if x and y are odd.
- (b) xy is even if and only if x and y are even.

(a) is true. If x and y are odd, then there are integers m and k such that $x = 2k + 1$ and $y = 2m + 1$. It follows that

$$\begin{aligned}xy &= (2k + 1)(2m + 1) = 4km + 2k + 2m + 1 \\ &= 2(2km + k + m) + 1\end{aligned}$$

is odd.

Conversely, by the contrapositive statement suppose that it is not true that both x and y are odd, that is, one of them is even. Assume without loss of generality that x is even. It follows that there is an integer k such that $x = 2k$, and thus $xy = 2ky$ is even.

(b) is false. If $x = 2$ and $y = 1$, then $xy = 2$ is even but $x = 2$ is even and $y = 1$ is odd.