Math 140
Introductory Statistics

Professor Bernardo Ábrego
Lecture 22
Sections 8.1

Back to Opinion Polls.

- At the beginning of this section, you read about a recent Phi Delta Kappa/Gallup poll that reported that a record 51% of the American public assigns a grade of A or B to the public schools in their community. The sample size was 1108. This survey had a margin of error of 3%, and so their 95% confidence interval is 48% to 54%. (The procedure Gallup uses to select a sample is more complicated than simple random sampling, but you can use your formula for a confidence interval to approximate Gallup's margin of error.)

You now should be able to answer these questions:

- What is it that you are 95% sure is in the confidence interval?

  Answer: The proportion of *all* Americans who would assign a grade of A or B to their local public schools.

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You now should be able to answer these questions:

- What is the interpretation of the confidence interval of 48% to 54%?

  Answer: We are 95% confident that if we could ask all Americans to give a grade to their local public schools, between 48% and 54% of them would give an A or B.

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You now should be able to answer these questions:

- What is the meaning of "95% confidence"?

  Answer: If we were to take 100 random samples of Americans and compute the 95% confidence interval from each sample, then we expect that 95 of them will contain the true proportion of all Americans that would assign a grade of A or B (whatever that proportion is).

What sample size should you use?

- Example. (p.426) Suppose you take a survey and get $\hat{p} = 0.7$. If your sample size is 100, what would be the margin of error for a 95% confidence interval?

$$E = z \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = (1.96)\sqrt{\frac{0.7(1-0.7)}{100}} = .0898$$

- If you quadruple your sample, what would be the new margin of error?

$$E = z \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = (1.96)\sqrt{\frac{0.7(1-0.7)}{400}} = .0449$$

What sample size should you use?

- Simple Answer: In general, the larger the sample the more accurate the results will be (smaller margin of error). If we would like to find the sample size for a particular margin of error, all we have to do is solve for n.

$$E = z \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$E^2 = z^2 \cdot \left(\frac{\hat{p}(1-\hat{p})}{n}\right)$$

$$\boxed{n = \frac{z^2 \cdot \hat{p}(1-\hat{p})}{E^2}}$$

If you do not have an estimate for $\hat{p}$, then you should use 0.5.

You may get a larger n than you need but never a smaller one.

Example: Estimating needed sample sizes (p.481)

- What sample size should you use for a survey if you want the margin of error to be at most 3% with 95% confidence but you have no estimate of p?

- D17. Suppose it costs $1 to survey each person in your sample. You judge that p is about 0.5. What will your survey cost if you want a margin of error of about 10%? 1%? 0.1%?

8.2 Testing a Proportion

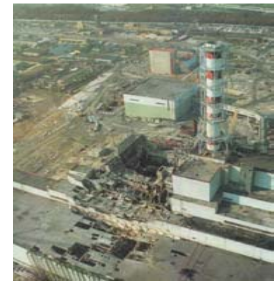
- People often make decisions with data by comparing the results from a sample to some predetermined standard. These kinds of decisions are called **tests of significance.**
- Goal**:** To test the significance of the difference between the sample and the standard.
  - Small difference: there is no reason to conclude that the standard doesn't hold.
  - Large enough difference: If it can't reasonably be attributed to chance, you can conclude that the standard no longer holds.

Example

- About 2% of barn swallows have white feathers in places where the plumage is normally blue or red. The white feathers are caused by genetic mutations.
- In 1986, the Russian nuclear reactor at Chernobyl leaked radioactivity. Researchers continue to be concerned that the radiation may have caused mutations in the genes of humans and animals that were passed on to offspring.

Example

- In a sample of barn swallows captured around Chernobyl in 1991 and 1996, about 14% had white feathers in places where the plumage is normally blue or red.
- Researchers compared the proportion 0.14 in the sample of captured barn swallows to the standard of .02. If the overall percentage was still only 2%, **it is not reasonably likely** to get 14% in their sample.
- So they came to the conclusion that there was an increased probability of genetic mutations in the Chernobyl area. Source: *Los Angeles Times*, October 9, 1997, page B2.



Informal Significance Testing

- People tend to believe that pennies are balanced. They generally have no qualms about flipping a penny to make a fair decision. Is it really the case that penny flipping is fair? What about spinning pennies?
- The logic involved in deciding whether or not to reject the standard that spinning a penny results in heads 50% of the time makes use of the same logic as that involved in estimating a proportion in Section 8.1.