Math 140
Introductory Statistics

Professor Bernardo Ábrego
Lecture 16
Sections 7.1,7.2

Notation

	Population	Sample	Sampling Distribution
Mean	μ	$\overline{x}$	$\mu_{\overline{x}}$
Standard Deviation	σ	s	$\sigma_{\overline{x}}$
Size	N	n	

Properties of The Sampling Distribution of The Sample Mean

- The mean $\mu_{\overline{x}}$ of the sampling distribution of $\overline{x}$ equals the mean of the population μ:

$$\mu_{\overline{x}} = \mu$$

- The standard deviation $\sigma_{\overline{x}}$ of the sampling distribution of $\overline{x}$, also called the **standard error** of the mean, equals the standard deviation of the population σ divided by the square root of the sample size n:

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$$

- The Shape of the sampling distribution will be approximately normal if the population is approximately normal; for other populations, the sampling distribution becomes more normal as n increases. This property is called the **Central Limit Theorem**.

Example 1

- Problems usually involve a combination of the three properties of the Sampling Distribution of the Sample Mean, together with what we learned about the normal distribution.

- Example: Average Number of Children
  What is the probability that a random sample of 20 families in the United States will have an average of 1.5 children or fewer?

Example 1

- Example: Average Number of Children

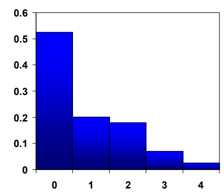
  What is the probability that a random sample of 20 families in the United States will have an average of 1.5 children or fewer?

Number of Children (per family), x	Proportion of families, $P(x)$
0	0.524
1	0.201
2	0.179
3	0.070
4 or more	0.026

- Mean (of population)

  $\mu = 0.873$
- Standard Deviation

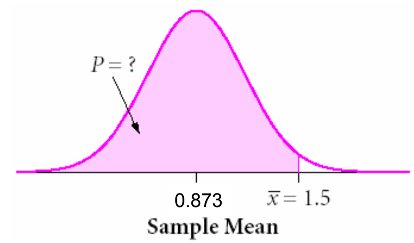
  $\sigma = 1.095$



Example 1

$$\mu_{\bar{x}} = \mu = 0.873$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.095}{\sqrt{20}} = 0.2448$$



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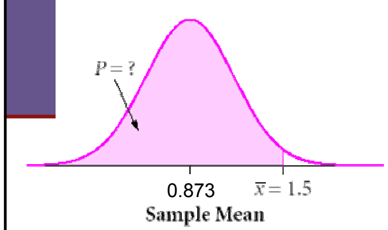

- Find z-score of the value 1.5

$$z = \frac{\bar{x} - mean}{SD} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{1.5 - 0.873}{0.2448}$$

$$\approx 2.56$$

$$\texttt{normalcdf}(-99999, 2.56) \approx .9947$$

- So in a random sample of 20 families there is a 99.47% probability that the mean number of children per family will be less than 1.5

Example 2

- Example: Reasonably Likely Averages

  What average numbers of children are reasonably likely in a random sample of 20 families?

- Recall that the values that are in the middle 95% of a random distribution are called **Reasonably Likely.**

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> Note that by calculating the z-scores of 2.5% and 97.5% we find that the Reasonably Likely values are those values within 1.96 standard deviations from the mean.
>
> That is, between $\mu - 1.96\ \sigma$ and $\mu + 1.96\ \sigma$

Finding Probabilities for Sample Totals

- Sometimes situations are stated in terms of the total number in the sample rather than the average number: "What is the probability that there are 30 or fewer children in a random sample of 20 families in the United States?" You have the choice of two equivalent ways to do this problem.
- **Method I:** Find the equivalent average number of children, $\overline{x}$, by dividing the total number of children, 30, by the sample size, 20:

$$\overline{x} = \frac{30}{20} = 1.5$$

Then you can use the same formulas and procedure as in the previous examples.

- **Method II:** Convert the formulas from the previous examples to equivalent formulas for the sum, then proceed as in the next example.

Sampling Distribution of the Sum of a Sample

- If a random sample of size n is selected with mean μ and standard deviation σ, then
  - the mean of the sampling distribution of the sum is

$$\mu_{sum} = n\mu$$

  - the standard error of the sampling distribution of the sum is

$$\sigma_{sum} = \sqrt{n} \cdot \sigma$$

  - the shape of the sampling distribution will be approximately normal if the population is approximately normal; for other populations, the sampling distribution becomes more normal as n increases.

> Note: To get the "sum" formulas just multiply by n

Examples 3 and 4

- Ex3: The Probability of 25 or fewer Children

  What is the probability that a random sample of 20 families in the United States will have a total of 25 children or fewer?

- Ex4: Reasonably Likely Totals

  In a random sample of 20 families, what total numbers of children are reasonably likely?