Math 140
Introductory Statistics

Professor Bernardo Ábrego
Lecture 11
Sections 5.1, 5.3

5.1 Models of Random Behavior

- Outcome: Result or answer obtained from a chance process.
- Event: Collection of outcomes.
- Probability: Number between 0 and 1 (0% and 100%). It tells how likely it is for an outcome or event to happen.
  - $P = 0$ The event cannot happen.
  - $P = 1$ The event is certain to happen.

5.1 Models of Random Behavior

- If the probability that event *A* happens is denoted *P*(*A*), then the probability that event *A* doesn't happen is *P*(*not A*) = 1 − *P*(*A*).
- The event *not A* is called the **complement** of event *A*.

Equally likely outcomes.

- If we have a list of all possible outcomes and **all of them are equally likely** then
- $P\,(\text{specific outcome}) = \dfrac{1}{\text{number of equally likely outcomes}}$
- $P\,(\text{event}) = \dfrac{\text{number of outcomes in event}}{\text{number of equally likely outcomes}}$
- Examples: Flipping a coin, rolling a fair die.

Equally likely outcomes.

- Jack and Jill, just won a contract to determine if people can tell tap water (T) from bottled water (B).
- They will give each person in their sample both kinds of water, in random order, and ask which is the tap water.
- Assuming that the tasters can't identify tap water, what is the probability that two tasters will guess correctly and choose *T*?

Tap vs Bottled Water.

Jack: There are three possible outcomes: Neither person chooses *T*, one chooses *T*, or both choose *T*. These three outcomes are equally likely, so each outcome has probability ⅓. In particular, the probability that both choose *T* is ⅓ .

Jill: Jack, did you break your crown already? I say there are *four* equally likely outcomes: Th e fi rst taster chooses *T* and the second also chooses *T* (*TT*); the first chooses *T* and the second chooses *B* (*TB*); the first chooses *B* and the second chooses *T* (*BT*); or both choose *B* (*BB*). Because these four outcomes are equally likely, each has probability ¼ . In particular, the probability that both choose *T* is ¼ , not ⅓.

Tap vs Bottled Water (Simulation).

- Jack and Jill use two flips of a coin to simulate the taste-test experiment with two tasters who can't identify tap water.
- Two tails represented neither person choosing the tap water, one heads and one tails represented one person choosing the tap water and the other choosing the bottled water, and two heads represented both people choosing the tap water.

Number Who Choose *T*	Frequency	Relative Frequency
0	782	0.26
1	1493	0.50
2	725	0.24
Total	**3000**	**1.00**

Display 5.2 Results of 3000 simulations for two tasters when $P(T) = 0.5$.

Law of Large Numbers

- In a random sampling, the larger the sample, the closer the proportion of successes in the sample tends to be the proportion in the population.
- Example, simulation of flipping a coin.

Number of Flips	10	100	1000	10000	100000
Heads	2	45	525	4990	50246
Tails	8	55	475	5010	49754

Sample Space

- A Sample Space for a chance process is a complete list of disjoint outcomes.
- Complete means that no possible outcomes are left off the list.
- Disjoint (or mutually exclusive) means that no two outcomes can occur at once.

- Often by symmetry we can assume that the outcomes on a sample space are equally likely. But to verify this we need to collect data and see if indeed each of the outcomes occurs the same number of times (approximately).

Examples

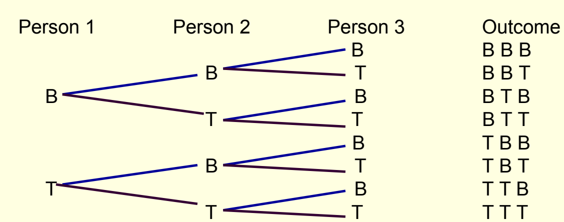
- Rolling a fair die.
  - Sample Space: {1,2,3,4,5,6}
  - P (4)= 1/6
  - P (number is even)= 3/6 = 1/2
- Selecting a card from a poker deck.
  - Sample Space: {A♥,2♥,3♥,…,Q♥,K♥,
    A♦,2♦,3♦,…,Q♦,K♦,
    A♣,2♣,3♣,…,Q♣,K♣,
    A♠,2♠,3♠,…,Q♠,K♠}

Examples

- Selecting a card from a poker deck.
  - Sample Space: {A♥,2♥,3♥,…,Q♥,K♥,
    A♦,2♦,3♦,…,Q♦,K♦,
    A♣,2♣,3♣,…,Q♣,K♣,
    A♠,2♠,3♠,…,Q♠,K♠}
  - P (3♥) = 1/52
  - P (Ace) = 4/52 = 1/13
  - P (♦) = 13/52 = 1/4

A random process is repeated several times

- To list the total list of outcomes when a random process is made up of many repetitions of another random process we can make a tree diagram.
- Example. Jack and Jill give samples of tap water (T) or bottled water (B) at random to three persons so that they taste it and see if they recognize tap water or not.



Fundamental Counting Principle

- For a two-stage process, with n_1 possible outcomes for stage 1 and n_2 possible outcomes for stage 2, the number of possible outcomes for the two stages together is $n_1 n_2$
- More generally, if there are k stages, with n_i possible outcomes for stage i, then the number of possible outcomes for all k stages taken together is $n_1 n_2 n_3 \ldots n_k$.

Discussion D8 (p. 296)

- Suppose you flip a fair coin five times.
  - a. How many possible outcomes are there?
  - b. What is the probability you get five heads?
  - c. What is the probability you get four heads and one tail?

5.3 Addition Rule and Disjoint Events

- First, "OR" in mathematics means one, the other, or **both**.
- Two events A and B are called **disjoint** (mutually exclusive) if they have no outcomes in common.
- If A and B are disjoint then

$$P(A \text{ or } B) = P(A) + P(B)$$

- Similarly if A, B, and C are mutually exclusive then

$$P(A \text{ or } B) = P(A) + P(B) + P(C)$$

Discussion *A* or *B*

Type of Fishing	**Number (Thousands)**
All freshwater fishing	31,041
Saltwater fishing	8,885
Total	**35,578**

- Are the categories in the table of Display 6.8 complete? Are they disjoint?
- What is the probability that a randomly selected person who fishes does their fishing in freshwater or in saltwater? How many people fish in both freshwater and saltwater?

Discussion A or B

Type of Fishing	Number (Thousands)
All freshwater fishing	31,041
Saltwater fishing	8,885
Total	**35,578**

- Are the categories in the table of Display 6.8 complete? Are they disjoint?
- Complete: **YES**, any person that fishes does so in either fresh water or salt water (maybe both)
- Disjoint: **NO**, the events “Saltwater” and “Freshwater” have outcomes in common.

Discussion A or B

Type of Fishing	Number (Thousands)
All freshwater fishing	31,041
Saltwater fishing	8,885
Total	**35,578**

- What is the probability that a randomly selected person who fishes does their fishing in freshwater or in saltwater? How many people fish in both freshwater and saltwater?
- P(“Fresh” or “Salt”) = 1

  However,

  P(“Fresh”) = $\frac{31041}{35578}$

  P(“Salt”) = $\frac{8885}{35578}$

and then,

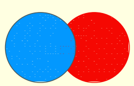
P(“Fresh”) + P(“Salt”) = $\frac{39926}{35578} > 1$

Discussion A or B

Type of Fishing	Number (Thousands)
All freshwater fishing	31,041
Saltwater fishing	8,885
Total	**35,578**

- What is the probability that a randomly selected person who fishes does their fishing in freshwater or in saltwater? How many people fish in both freshwater and saltwater?

Fresh Water



Salt Water

(“Only Salt”) + #(“Fresh”) = 35578
(“Only Salt”) + 31041 = 35578
(“Only Salt”) = 35578 – 31041 = **4537**

Similarly
(“Only Fresh”) = 35578 – 8885
(“Only Fresh”) = **26693**.

(“Only Salt” or “Only Fresh”) = **31230**

Then,
#(“Fresh” and “Salt”) = 35578 – 31230
#(“Fresh” and “Salt”) = **4348**