

MANOVA

Lecture 12

Nuance stuff

Psy 524

Andrew Ainsworth



0011

Multivariate Analysis of Covariance

0011

- The linear combination of DVs is adjusted for one or more Covariates.
- The adjusted linear combinations of the DVs is the combination that would have been had all of the subjects scored the same on the CVs.

$$S^* = S^{(Y)} - S^{(YZ)} (S^{(Z)})^{-1} S^{(ZY)}$$

Multivariate Analysis of Covariance

0011

- Each subjects score is made up of the DVs and the CVs

$$Y_{111} = \begin{bmatrix} 110 \\ 115 \\ 108 \end{bmatrix} \begin{bmatrix} IQ \\ wrat - r \\ wrat - a \end{bmatrix}$$

Multivariate Analysis of Covariance

0011

- So that each S is a combination of the original S plus the SSCP for the CVs and the covariances between the DVs and the CVs.

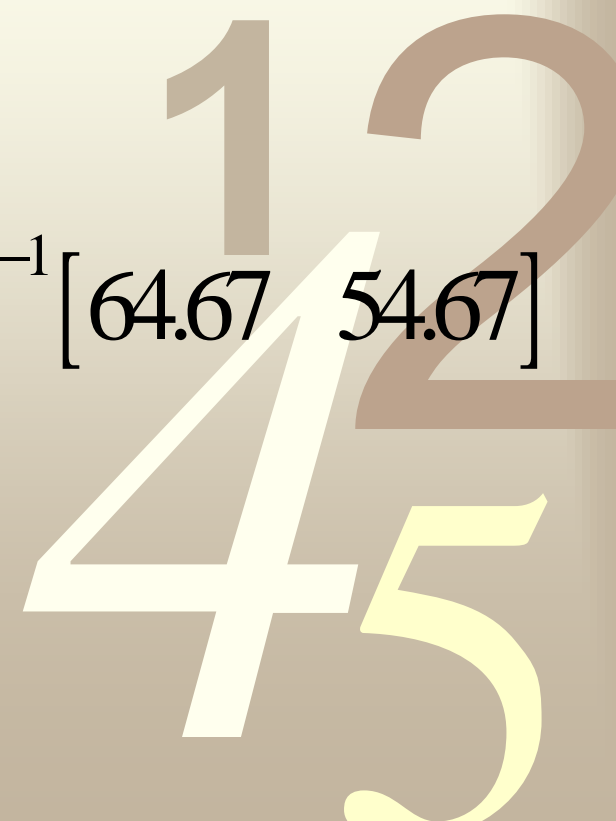
	Z	Y_1	Y_2
Z	$[2.00]$	$[64.67$	$54.67]$
Y_1	$[64.67]$	2090.89	1767.89
Y_2	$[54.67]$	1767.56	1494.22

Multivariate Analysis of Covariance

0011

$$S^* = S^{(Y)} - S^{(YZ)} (S^{(Z)})^{-1} S^{(ZY)}$$

$$S^* = \begin{bmatrix} 2090.89 & 1767.56 \\ 1767.56 & 1494.22 \end{bmatrix} - \begin{bmatrix} 64.67 \\ 54.67 \end{bmatrix} [2]^{-1} \begin{bmatrix} 64.67 & 54.67 \end{bmatrix}$$



Multivariate Analysis of Covariance

0011

- Calculating Wilk's Lambda is the same and for the most part the F-test is the same except calculating s and DF2:

$$s = \sqrt{\frac{(p+q)^2 (df_{effect})^2 - 4}{(p+q)^2 + (df_{effect})^2 - 5}}$$
$$df_2 = s \left[(df_{error}) - \frac{(p+q) - df_{error} + 1}{2} \right] - \left[\frac{(p+q)(df_{error}) - 2}{2} \right]$$

Different Multivariate test criteria

0011

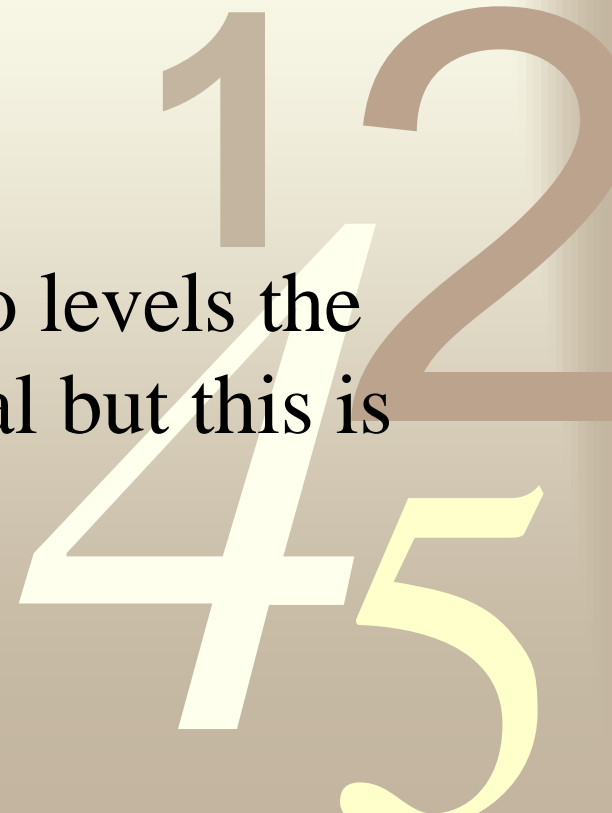
- Hotelling's Trace
- Wilk's Lambda,
- Pillai's Trace
- Roy's Largest Root



Different Multivariate test criteria

0011

- When there are only two levels for an effect $s=1$ and all of the tests should be identical
- When there are more than two levels the tests should be nearly identical but this is not always the case



Different Multivariate test criteria

0011

- When there are more than two levels there are multiple ways in which the data can be combined to separate the groups
 - (e.g. one dimension separates group 1 from groups 2 and 3, a second dimension separates group 2 from group 3, etc.)



Different Multivariate test criteria

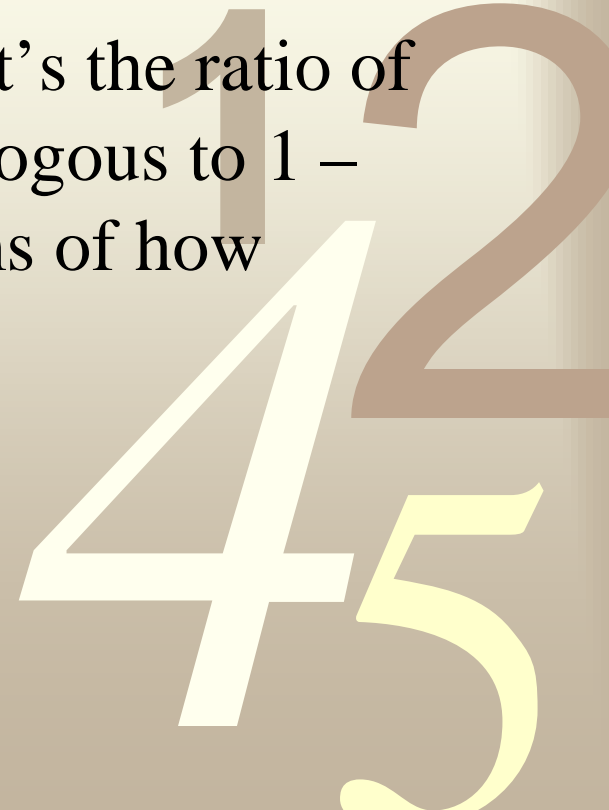
0011

- Wilk's Lambda, Hotelling's Trace and Pillai's trace all pool the variance from all the dimensions to create the test statistic.
- Roy's largest root only uses the variance from the dimension that separates the groups most (the largest "root" or difference).

Different Multivariate test criteria

0011

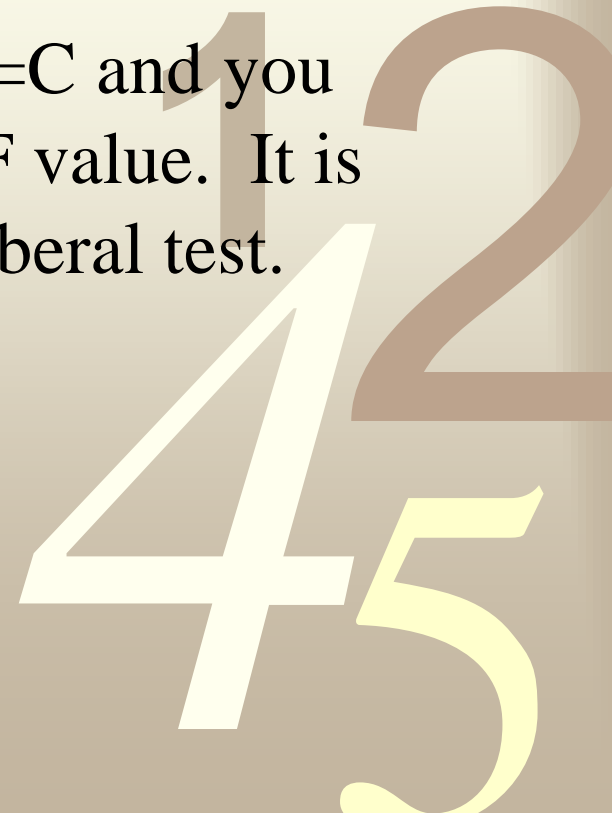
- The various formulas are (E is error and H is hypothesized effect):
 - Wilk's Lambda - $|E| / |H + E|$ - It's the ratio of error to effect plus error. Analogous to $1 - R^2$. Middle of the road in terms of how conservative a test it is.



Different Multivariate test criteria

0011

- The various formulas are (E is error and H is hypothesized effect):
 - Hotelling's trace – $\text{Trace}(H/E)=C$ and you look up C in a table to get the F value. It is analogous to an F-test. Very liberal test.



Different Multivariate test criteria

0011

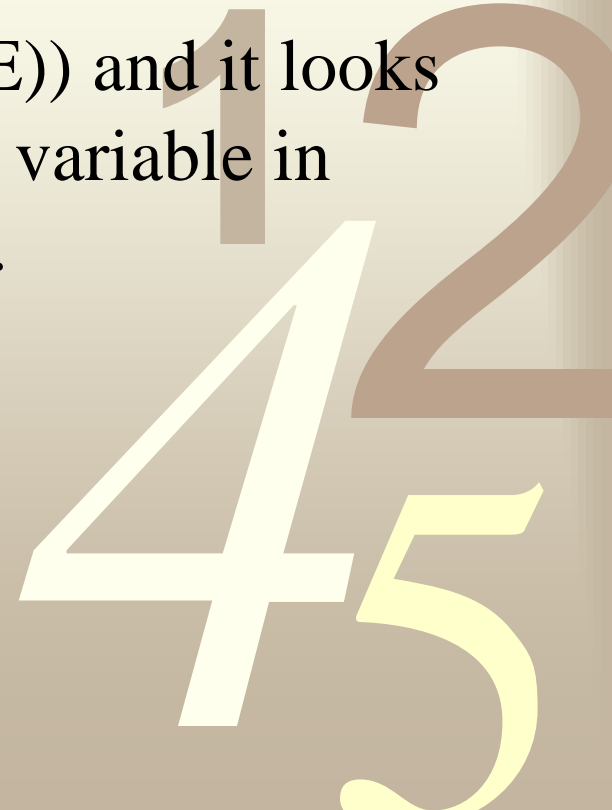
- The various formulas are (E is error and H is hypothesized effect):
 - Pillai's trace – $\text{Trace}(H/(H + E))$. Analogous to R^2 . Very conservative



Different Multivariate test criteria

0011

- The various formulas are (E is error and H is hypothesized effect):
 - Roy's Largest Root - $(H/(H + E))$ and it looks for the biggest difference. It is variable in terms of how conservative it is.

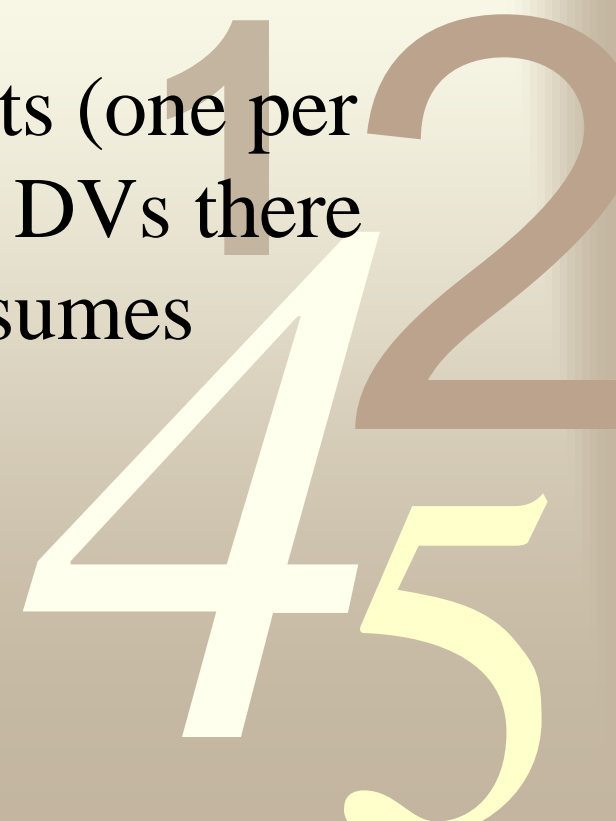


Which do you choose?

- For the most part stick with Wilk's lambda. It's the most widely used
- Use Hotelling's Trace if
 - Manipulated (experimental) variables
 - Very clean design with no internal validity problems
- Pillai's trace is the most conservative, but if your design has many problems (e.g. unbalanced, assumption violation, etc) pillai's is supposed to be robust to these problems

Assessing DVs

- If multivariate test is significant
- Run multiple univariate F-tests (one per DV) in order to see on which DVs there are group differences, this assumes uncorrelated DVs.



Assessing DVs

- The overall alpha level should be controlled for considering the multiple tests

$$\mathbf{a}_{overall} = 1 - (1 - \mathbf{a}_1)(1 - \mathbf{a}_2) \dots (1 - \mathbf{a}_p)$$

- The alpha levels can be divided equally or they can be set up to give more important tests a more liberal alpha level.

Assessing DVs

- If DVs are correlated than individual F-tests are problematic but usually this is ignored and univariate Fs interpreted anyway



Assessing DVs

0011

- Roy-Bargman step down procedure
 - Can be used as follow-up to MANOVA or MANCOVA with correlated DVs or as alternative to multivariate analysis all together.

12
45

Assessing DVs

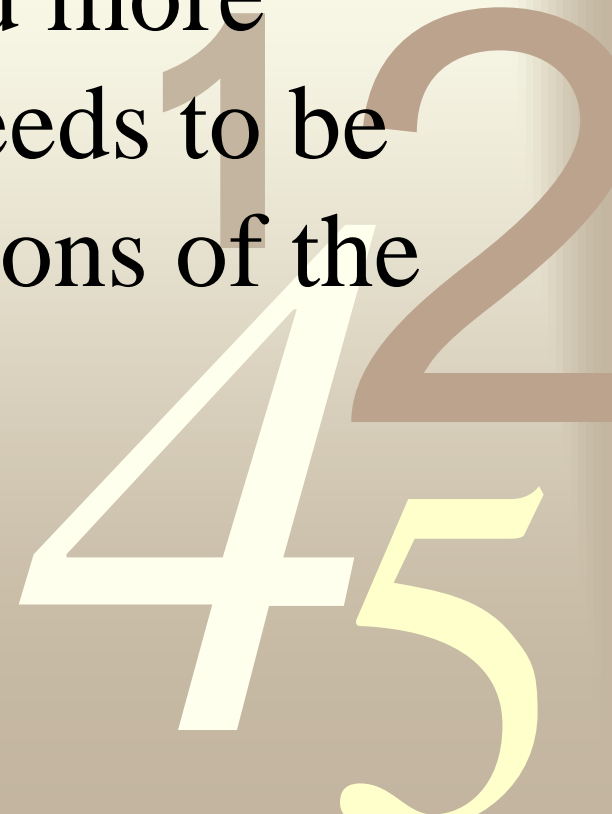
- Roy-Bargman step down procedure
 - The theoretically most important DV is analyzed as an individual univariate test (DV1).
 - The next DV (DV2), in terms of theoretical importance, is then analyzed using DV1 as a covariate. This controls for the relationship between the two DVs.
 - DV3 (in terms of importance) is assessed with DV1 and DV2 as covariates, etc.

Assessing DVs

- Discriminant Function analysis –
 - We will discuss this more later but...
 - It uses group membership as the DV and the MANOVA DVs as predictors of group membership
 - Using this as a follow up to MANOVA will give you the relative importance of each DV predicting group membership (in a multiple regression sense)

Specific Comparisons and Trend Analysis

- With a significant multivariate (and univariate) test and more than two groups, this needs to be followed with comparisons of the individual groups.



Specific Comparisons and Trend Analysis

- Just like any test discussed previously, this can be done with planned or post hoc comparisons.
- Planned comparisons can be written into SPSS syntax and if post hoc you can adjust the test by the degrees of freedom to get a Scheffe adjustment.

Unequal samples

- If intended to be equal and no meaning to the imbalance, use type 3 sums of squares
- If the imbalance is meaningful use type 1 sums of squares

