MANOVA
Lecture 12

Nuance stuff

Psy 524

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Multivariate Analysis of Covariance

- The linear combination of DVs is adjusted for one or more Covariates.
- The adjusted linear combinations of the DVs is the combination that would have been had all of the subjects scored the same on the CVs.

$$S^{*} = S^{(Y)} - S^{(YZ)}(S^{(Z)})^{-1}S^{(ZY)}$$

Multivariate Analysis of Covariance

- Each subjects score is made up of the DVs and the CVs

$$Y_{111} = \begin{bmatrix} 110 \\ 115 \\ 108 \end{bmatrix} \begin{bmatrix} IQ \\ wrat-r \\ wrat-a \end{bmatrix}$$

Multivariate Analysis of Covariance

- So that each S is a combination of the original S plus the SSCP for the CVs and the covariances between the DVs and the CVs.

$$
\begin{array}{cc}
 & \begin{array}{ccc} Z & \quad\quad Y_1 & \quad\quad Y_2 \end{array} \\
\begin{array}{c} Z \\ Y_1 \\ Y_2 \end{array} &
\left[\begin{array}{c|cc}
[2.00] & [64.67 & 54.67] \\
\hline
\left[\begin{array}{c} 64.67 \\ 54.67 \end{array}\right. & 2090.89 & 1767.89 \\
\left.\right] & 1767.56 & 1494.22
\end{array}\right]
\end{array}
$$

Multivariate Analysis of Covariance

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$$S^{*} = S^{(Y)} - S^{(YZ)} (S^{(Z)})^{-1} S^{(ZY)}$$

$$S^{*} = \begin{bmatrix} 2090.89 & 1767.56 \\ 1767.56 & 1494.22 \end{bmatrix} - \begin{bmatrix} 64.67 \\ 54.67 \end{bmatrix} [2]^{-1} \begin{bmatrix} 64.67 & 54.67 \end{bmatrix}$$

Multivariate Analysis of Covariance

- Calculating Wilk's Lambda is the same and for the most part the F-test is the same except calculating *s* and DF2:

$$s = \sqrt{\frac{(p+q)^2 (df_{effect})^2 - 4}{(p+q)^2 + (df_{effect})^2 - 5}}$$

$$df_2 = s\left[(df_{error}) - \frac{(p+q) - df_{error} + 1}{2}\right] - \left[\frac{(p+q)(df_{error}) - 2}{2}\right]$$

Different Multivariate test criteria

- Hotelling’s Trace
- Wilk’s Lambda,
- Pillai’s Trace
- Roy’s Largest Root

Different Multivariate test criteria

- When there are only two levels for an effect s=1 and all of the tests should be identical

- When there are more than two levels the tests should be nearly identical but this is not always the case

Different Multivariate test criteria

- When there are more than two levels there are multiple ways in which the data can be combined to separate the groups
  - (e.g. one dimension separates group 1 from groups 2 and 3, a second dimension separates group 2 from group 3, etc.)

Different Multivariate test criteria

- Wilk’s Lambda, Hotelling’s Trace and Pillai’s trace all pool the variance from all the dimensions to create the test statistic.

- Roy’s largest root only uses the variance from the dimension that separates the groups most (the largest “root” or difference).

Different Multivariate test criteria

- The various formulas are (E is error and H is hypothesized effect):
  - Wilk’s Lambda - |E| |H + E| - It’s the ratio of error to effect plus error. Analogous to 1 – R2. Middle of the road in terms of how conservative a test it is.

Different Multivariate test criteria

- The various formulas are (E is error and H is hypothesized effect):
  - Hotelling’s trace – Trace(H/E)=C and you look up C in a table to get the F value. It is analogous to an F-test. Very liberal test.

Different Multivariate test criteria

- The various formulas are (E is error and H is hypothesized effect):
  - Pillai's trace – Trace(H/(H + E)). Analogous to R2. Very conservative

Different Multivariate test criteria

- The various formulas are (E is error and H is hypothesized effect):
  - Roy's Largest Root - (H/(H + E)) and it looks for the biggest difference. It is variable in terms of how conservative it is.

Which do you choose?

- For the most part stick with Wilk's lambda. It's the most widely used
- Use Hotelling's Trace if
  - Manipulated (experimental) variables
  - Very clean design with no internal validity problems
- Pillai's trace is the most conservative, but if your design has many problems (e.g. unbalanced, assumption violation, etc) pillai's is supposed to be robust to these problems

Assessing DVs

- If multivariate test is significant
- Run multiple univariate F-tests (one per DV) in order to see on which DVs there are group differences, this assumes uncorrelated DVs.

Assessing DVs

- The overall alpha level should be controlled for considering the multiple tests

$$a_{overall} = 1 - (1 - a_1)(1 - a_2)\ldots(1 - a_p)$$

- The alpha levels can be divided equally or they can be set up to give more important tests a more liberal alpha level.

Assessing DVs

- If DVs are correlated than individual F-tests are problematic but usually this is ignored and univariate Fs interpreted anyway

Assessing DVs

- Roy-Bargman step down procedure
  - Can be used as follow-up to MANOVA or MANCOVA with correlated DVs or as alternative to multivariate analysis all together.

Assessing DVs

- Roy-Bargman step down procedure
  - The theoretically most important DV is analyzed as an individual univariate test (DV1).
  - The next DV (DV2), in terms of theoretical importance, is then analyzed using DV1 as a covariate. This controls for the relationship between the two DVs.
  - DV3 (in terms of importance) is assessed with DV1 and DV2 as covariates, etc.

Assessing DVs

- Discriminant Function analysis –
  - We will discuss this more later but…
  - It uses group membership as the DV and the MANOVA DVs as predictors of group membership
  - Using this as a follow up to MANOVA will give you the relative importance of each DV predicting group membership (in a multiple regression sense)

Specific Comparisons and Trend Analysis

- With a significant multivariate (and univariate) test and more than two groups, this needs to be followed with comparisons of the individual groups.

Specific Comparisons and Trend Analysis

- Just like any test discussed previously, this can be done with planned or post hoc comparisons.
- Planned comparisons can be written into SPSS syntax and if post hoc you can adjust the test by the degrees of freedom to get a Scheffe adjustment.

Unequal samples

- If intended to be equal and no meaning to the imbalance, use type 3 sums of squares
- If the imbalance is meaningful use type 1 sums of squares