Repeated Measures ANOVA
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Ψ320
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Major Topics
- What are repeated-measures?
- An example
- Assumptions
- Advantages and disadvantages
- Effect size

Repeated Measures?
- Between-subjects designs
  - different subjects serve in different treatment levels
  - (what we already know)
- Repeated-measures (RM) designs
  - each subject receives all levels of at least one independent variable
  - (what we’re learning today)
Repeated Measures

- All subjects get all treatments.
- All subjects receive all levels of the independent variable.
- Different n’s are unusual and cause problems (i.e. dropout or mortality).
- Treatments are usually carried out one after the other (in serial).

Example: Counseling For PTSD

- Foa, et al. (1991)
  - Provided supportive counseling (and other therapies) to victims of rape
  - Do number of symptoms change with time?
    - There’s no control group for comparison
    - Not a test of effectiveness of supportive counseling

Example: Counseling For PTSD

- 9 subjects measured before therapy, after therapy, and 3 months later
- We are ignoring Foa’s other treatment conditions.
Example: Counseling For PTSD

- Dependent variable = number of reported symptoms.
- Question: Do number of symptoms decrease over therapy and remain low?
- Data on next slide

The Data

<table>
<thead>
<tr>
<th>Patient</th>
<th>Pre</th>
<th>Post</th>
<th>Follow-Up</th>
<th>Subject Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21</td>
<td>15</td>
<td>15</td>
<td>17.000</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td>15</td>
<td>8</td>
<td>15.667</td>
</tr>
<tr>
<td>3</td>
<td>21</td>
<td>17</td>
<td>22</td>
<td>20.000</td>
</tr>
<tr>
<td>4</td>
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<td>20.333</td>
</tr>
<tr>
<td>5</td>
<td>32</td>
<td>17</td>
<td>16</td>
<td>21.667</td>
</tr>
<tr>
<td>6</td>
<td>27</td>
<td>20</td>
<td>17</td>
<td>21.333</td>
</tr>
<tr>
<td>7</td>
<td>21</td>
<td>8</td>
<td>8</td>
<td>12.333</td>
</tr>
<tr>
<td>8</td>
<td>25</td>
<td>19</td>
<td>15</td>
<td>19.667</td>
</tr>
<tr>
<td>9</td>
<td>18</td>
<td>10</td>
<td>3</td>
<td>10.333</td>
</tr>
</tbody>
</table>

Mean 23.889 15.667 13.222 17.593
SD 4.197 4.243 5.783 4.072

Preliminary Observations

- Notice that subjects differ from each other.
  - Between-subjects variability
- Notice that means decrease over time
  - Faster at first, and then more slowly
  - Within-subjects variability
Partitioning Variability

This partitioning is reflected in the summary table.

Sums of Squares

The total variability can be partitioned into Between Groups (e.g. measures), Subjects and Error Variability

\[
\sum (Y_i - \bar{Y}_{GM})^2 = \sum n_i (\bar{Y}_i - \bar{Y}_{GM})^2 + g \sum (\bar{Y}_j - \bar{Y}_{GM})^2 + \\
+ \left[ \sum (Y_i - \bar{Y}_j)^2 - g \sum (\bar{Y}_j - \bar{Y}_{GM})^2 \right]
\]

\[
SS_{Total} = SS_{BetweenGroups} + SS_{Subjects} + SS_{Error}
\]

Deviation Sums of Squares

\[
SS_{Total} = \sum (Y_i - \bar{Y}_{GM})^2 = \\
(26 - 17.593)^2 + (32 - 17.593)^2 + (27 - 17.593)^2 + \\
(21 - 17.593)^2 + (25 - 17.593)^2 + (18 - 17.593)^2 + \\
(15 - 17.593)^2 + (15 - 17.593)^2 + (17 - 17.593)^2 + \\
+ \cdots + (8 - 17.593)^2 + (15 - 17.593)^2 + (3 - 17.593)^2 = \\
= 1114.519
\]
Deviation Sums of Squares

\[ SS_{BG} = \sum n_j \left( \bar{Y}_j - \bar{Y}_{GM} \right)^2 = \]

\[ = \[\] \times (\cdot - \cdot)^2 + \] \times (\cdot - \cdot)^2 + \] \times (13.222 - 17.593)^2 = 562.074

Deviation Sums of Squares

\[ SS_{subject} = g \sum (\bar{Y}_j - \bar{Y}_{GM})^2 = \]

\[ = \[\] \times (\cdot - \cdot)^2 + \] \times (\cdot - \cdot)^2 + \] \times (20.333 - 17.593)^2 + \] \times (21.667 - 17.593)^2 + \] \times (12.333 - 17.593)^2 + \] \times (19.667 - 17.593)^2 + \] \times (10.333 - 17.593)^2 = 397.852

Deviation Sums of Squares

\[ SS_{error} = \left[ \sum (Y_{ij} - \bar{Y}_j)^2 - g \sum (Y_{ij} - \bar{Y}_{GM})^2 \right] = \]

\[ \sum (Y_{ij} - \bar{Y}_j)^2 = (\cdot - \cdot)^2 + (\cdot - \cdot)^2 + \]

\[ + (26 - 23.889)^2 + (32 - 23.889)^2 + (27 - 23.889)^2 + \]

\[ + (21 - 23.889)^2 + (25 - 23.889)^2 + (18 - 23.889)^2 + \]

\[ + (15 - 15.667)^2 + (15 - 15.667)^2 + (17 - 15.667)^2 + \]

\[ + \cdots + (8 - 13.222)^2 + (15 - 13.222)^2 + (3 - 13.222)^2 = 552.444 \]

\[ = 552.444 - 397.852 = 154.592 \]
### Computational Approach

<table>
<thead>
<tr>
<th>Patient</th>
<th>Pre</th>
<th>Post</th>
<th>Follow-Up</th>
<th>Subject Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21</td>
<td>15</td>
<td>15</td>
<td>51</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td>15</td>
<td>8</td>
<td>47</td>
</tr>
<tr>
<td>3</td>
<td>21</td>
<td>17</td>
<td>22</td>
<td>60</td>
</tr>
<tr>
<td>4</td>
<td>26</td>
<td>20</td>
<td>15</td>
<td>61</td>
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<td>5</td>
<td>22</td>
<td>17</td>
<td>16</td>
<td>65</td>
</tr>
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<td>6</td>
<td>27</td>
<td>20</td>
<td>17</td>
<td>64</td>
</tr>
<tr>
<td>7</td>
<td>21</td>
<td>8</td>
<td>8</td>
<td>37</td>
</tr>
<tr>
<td>8</td>
<td>25</td>
<td>19</td>
<td>15</td>
<td>59</td>
</tr>
<tr>
<td>9</td>
<td>18</td>
<td>10</td>
<td>3</td>
<td>31</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td><strong>215</strong></td>
<td><strong>141</strong></td>
<td><strong>119</strong></td>
<td><strong>475</strong></td>
</tr>
<tr>
<td>$\Sigma Y^2$</td>
<td></td>
<td></td>
<td></td>
<td>9471</td>
</tr>
</tbody>
</table>

### Computational Sums of Squares

\[
SS_{bg} = \frac{\sum (\sum A_i)^2}{s} - \frac{T^2}{as}
\]

\[
SS_{Subject} = \frac{\sum (\sum S_i)^2}{d} - \frac{T^2}{as}
\]

\[
SS_{Error} = \sum Y^2 - \frac{\sum (\sum A_i)^2}{s} - \frac{\sum (\sum S_i)^2}{d} + \frac{T^2}{as}
\]

\[
SS_{Total} = \sum Y^2 - \frac{T^2}{as}
\]

### Computational SS Example

\[
SS_A = \frac{2^2 + 2^2 + 119^2}{3} - \frac{121}{9} = 8356.481 - \frac{121}{9} = 562.075
\]

\[
SS_A = \frac{2^2 + 2^2 + 61^2 + 65^2 + 64^2 + 37^2 + 59^2 + 31^2}{8} - 8356.481
\]

\[
SS_A = \frac{8356.481}{8} - 8356.481 = 397.852
\]

\[
SS_{Error} = 9471 - 8918.556 - 8754.333 + 8356.481 = 154.592
\]

\[
SS_T = 9471 - 8356.481 = 1114.519
\]
Degrees of Freedom

\[
\begin{align*}
\text{From BG ANOVA} \\
& \quad \begin{aligned}
& \text{df}_{bg} = g - 1 = 3 - 1 = 2 \\
& \text{df}_{ws} = g (n - 1) = N - g = 27 - 3 = 24 \\
& \rightarrow \text{df}_i = n - 1 = 9 - 1 = 8 \\
& \rightarrow \text{df}_{error} = (n - 1) * (g - 1) = 8 * 2 = 16 \\
& \text{df}_{total} = N - 1 = 27 - 1 = 26
\end{aligned}
\end{align*}
\]

---

Summary Table

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>562.074</td>
<td>2</td>
<td>261.037</td>
<td>29.087</td>
</tr>
<tr>
<td>WS (from BG design)</td>
<td>552.444</td>
<td>24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subject</td>
<td>397.852</td>
<td>8</td>
<td>49.731</td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>154.593</td>
<td>16</td>
<td>9.662</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1114.519</td>
<td>26</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*\( F_{crit} \) with 2 and 16 degrees of freedom is 3.63 we would reject \( h_0 \)

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Plot of the Data

![Plot of the Data](image-url)
Interpretation

- Note parallel with diagram
- Note subject differences not in error term
- Note $\text{MS}_{\text{error}}$ is denominator for $F$ on Time
- Note $\text{SS}_{\text{time}}$ measures what we are interested in studying

Assumptions

- Correlations between trials are all equal
  - Actually more than necessary, but close
  - Matrix shown below

<table>
<thead>
<tr>
<th></th>
<th>Pre</th>
<th>Post</th>
<th>Followup</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre</td>
<td>1.00</td>
<td>.637</td>
<td>.434</td>
</tr>
<tr>
<td>Post</td>
<td>1.00</td>
<td>.742</td>
<td></td>
</tr>
<tr>
<td>Followup</td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
</tbody>
</table>

- Previous matrix might look like we violated assumptions
  - Only 9 subjects
  - Minor violations are not too serious.
- Greenhouse and Geisser (1959) correction (among many)
  - Adjusts degrees of freedom
Multiple Comparisons

- With few means:
  - t-test with Bonferroni corrections
  - Limit to important comparisons
- With more means:
  - Require specialized techniques
  - Trend analysis

Advantages of RM Designs

- Eliminate subject differences from error term
  - Greater power
- Fewer subjects needed
- Often only way to address the problem
  - This example illustrates that case.

Disadvantages of RM designs

- Carry-over effects
  - Counter-balancing
- May tip off subjects to what you are testing
- Some phenomenon cannot be tested in a repeated measure fashion (e.g. anything that requires tricking the participant)
Effect Size

- Simple extension of what we said for $t$ test for related samples.
- Stick to pairs of means.
- OR
  - $\eta^2$ can be used for repeated measures data as well
  - Some adjustments can make it more meaningful

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