# Hypothesis Tests: One Sample Mean

**Cal State Northridge**
**Ψ320**
**Andrew Ainsworth PhD**

## Major Points

- Sampling distribution of the mean revisited
- Testing hypotheses: sigma known
  - An example
- Testing hypotheses: sigma unknown
  - An example
- Factors affecting the test
- Measuring the size of the effect
- Confidence intervals



## Review: Hypothesis Testing Steps

1. State Null Hypothesis
2. Alternative Hypothesis
3. Decide on α (usually .05)
4. Decide on type of test (distribution; *z*, *t*, etc.)
5. Find critical value & state decision rule
6. Calculate test
7. Apply decision rule

## Sampling Distributions

- In reality, we take only ***one*** sample of a specific size (*N*) from a population and calculate a statistic of interest.
- Based upon this single statistic from a single sample, we want to know:
  - *"How likely is it that I could get a sample statistic of this value from a population if the corresponding population parameter was ___"*

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## Sampling Distributions

- BUT, in order to answer that question, we need to know what the entire range of values this statistic *could* be.
- How can we find this out?
- Draw *all possible* samples of size *N* from the population and calculate a sample statistic on *each of these samples (Chapter 8)*
- **Or** we can calculate it

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## Sampling Distributions

- A distribution of *all possible* statistics calculated from *all possible* samples of size *N* drawn from a population is called a **Sampling Distribution.**
- Three things we want to know about *any* distribution?
  - – Central Tendency, Dispersion, Shape

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## An Example – Back to IQPLUS

- Returning to our study of IQPLUS and its affect on IQ
- A group of 25 participants are given 30mg of IQPLUS everyday for ten days
- At the end of 10 days the 25 participants are given the Stanford-Binet intelligence test.

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## IQPLUS

- The mean IQ score of the 25 participants is 106

  $\mu = 100, \sigma = 15$
- Is this increase large enough to conclude that IQPLUS was affective in increasing the participants IQ?

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## Sampling Distribution of the Mean

- Formal solution to example given in Chapter 8.
- We need to know what kinds of sample means to expect if IQPLUS has no effect.
  - i. e. What kinds of means if $\mu = 100$ and $\sigma = 15$?
  - This is the sampling distribution of the mean (Why?)

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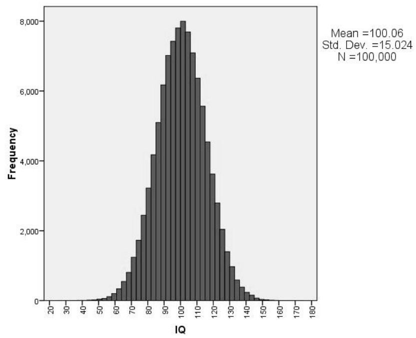
## Population Distribution

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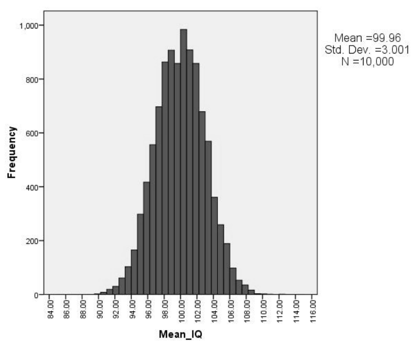
## Sampling Distribution




What is the relationship between σ and the SD above?

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## Sampling Distribution of the Mean

- The sampling distribution of the mean depends on
  - Mean of sampled population
    - Why?
  - St. dev. of sampled population
    - Why?
  - Size of sample
    - Why?

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## Sampling Distribution of the mean

- Shape of the sampling distribution
  - Approaches normal
    - Why?
  - Rate of approach depends on sample size
    - Why?
- Basic theorem
  - Central limit theorem

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## Central Limit Theorem

- **Central Tendency**
  - The mean of the Sampling Distribution of the mean is denoted as $\mu_{\bar{X}}$
- **Dispersion**
  - The Standard Deviation of the Sampling Distribution of the mean is called the **Standard Error of the Mean** and is denoted as $\sigma_{\bar{X}}$

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## Central Limit Theorem

- Standard Error of the Mean
  - We defined this manually in Chapter 8
  - And it can be calculated as:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

- Shape
  - The shape of the sampling distribution of the mean will be normal if the original population is normally distributed ***OR***
  - if the sample size is "reasonably large."

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## DEMONSTRATION

- Let a population be very skewed
- Draw samples of size 3 and calculate means
- Draw samples of size 10 and calculate means
- Plot means
- Note changes in means, standard deviations, and shapes

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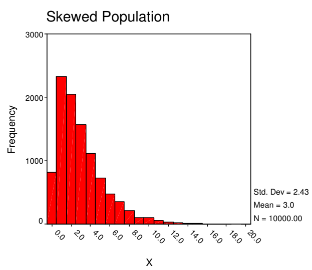
## PARENT POPULATION

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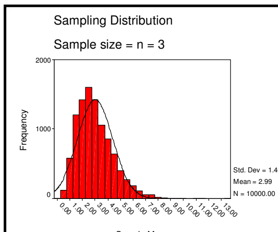


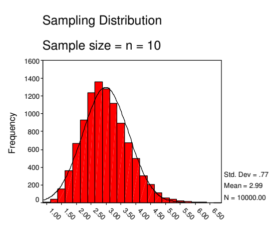

## Demonstration

- Means have stayed at 3.00 throughout
  - Except for minor sampling error
- Standard deviations have decreased appropriately
- Shape has become more normal as we move from n = 3 to n = 10
  - See superimposed normal distribution for reference

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## Testing Hypotheses: μ and σ known

Called a 1-sample Z-test

- $H_0$: $\mu = 100$
- $H_1$: $\mu \neq 100$ (Two-tailed)
- Calculate *p* (sample mean) = 106 if $\mu = 100$
- Use *z* from normal distribution
- Sampling distribution would be normal

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## Using z to Test $H_0 \rightarrow$ 2-tailed$_{\alpha = .05}$

- Calculate *z*

$$z_{\bar{X}} = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} = \frac{\_\_\_ - \_\_\_}{\_\_ / \sqrt{\_\_}} = \frac{\_\_}{\_\_} = \_\_\_$$

- *If* z > $\pm$ 1.96, reject $H_0$ (Why 1.96?)
- ____ > 1.96
  - The difference is significant.

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## USING Z TO TEST $H_0 \rightarrow$ 1-TAILED$_{\alpha = .05}$

- Calculate $z$ (from last slide)
- *If* z > $\pm$ 1.64, reject $H_0$ (Why 1.64?)
- ____ > 1.64
  - The difference is significant.

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## Z-TEST

- Compare computed $z$ to histogram of sampling distribution
- The results should look consistent.
- Logic of test
  - Calculate probability of getting this mean if null true.
  - Reject if that probability is too small.

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## TESTING HYPOTHESES: $\mu$ KNOWN $\sigma$ NOT KNOWN

- Assume same example, but $\sigma$ not known
- We can make a guess at $\sigma$ with $s$
- But, unless we have a large sample, $s$ is likely to underestimate $\sigma$ (see next slide)
- So, a test based on the normal distribution will lead to biased results (e.g. more Type 1 errors)

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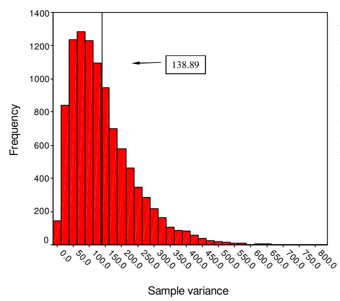
## Sampling Distribution of the Variance




Let's say you have a population variance = 138.89

If $n = 5$ and you take 10,000 samples

58.94% < 138.89

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## Testing Hypotheses: μ known σ not known

- Since $s$ is the best estimate of σ; $s_{\bar{X}}$ is the best estimate of $\sigma_{\bar{X}}$
- Since Z does not work in this case we need a different distribution
  - One that is based on $s$
  - Adjusts for the underestimation
  - And takes sample size (i.e. *degrees of freedom*) into account

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## The *t* Distribution

- Symmetric, mean = median = mode = 0.
- Asymptotic tails
- Infinite family of $t$ distributions, one for every possible $df$ .
  - For low $df$, the t distribution is more leptokurtic (e.g. spiked, thin, w/ fat tails)
  - For high $df$, the t distribution is more normal
  - With $df = \infty$, the t distribution and the z distribution are equivalent.

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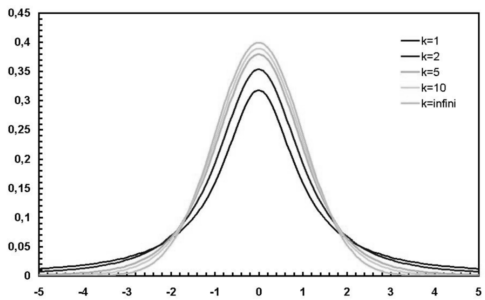
## The *t* Distribution

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## Degrees of Freedom

- Skewness of sampling distribution of variance decreases as $n$ increases
- $t$ will differ from $z$ less as sample size increases
- Therefore need to adjust $t$ accordingly
- *Degrees of Freedom:* $df = n - 1$
- $t$ based on $df$

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## Testing Hypotheses: μ known σ not known

Called a 1-sample $t$-test

- $H_0$: $\mu = 100$
- $H_1$: $\mu \neq 100$ (Two-tailed)
- Calculate $p$ (sample mean) = 106 if $\mu = 100$
- Use $t$-table to look up critical value using *degrees of freedom*
- Compare $t_{observed}$ to $t_{critical}$ and make decision

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## USING *T* TO TEST $H_0 \rightarrow$ 2-TAILED$_{\alpha\,=\,.05}$

- Same as $z$ except for $s$ in place of $\sigma$.
- Let's say for the 25, $s$ = 7.78

$$t_{observed} = \frac{\overline{X} - \mu}{s_{\overline{X}}} = \frac{\_\_\_ - \_\_\_}{\_\_\_ / \sqrt{\_\_}} = \frac{\_\_}{\_\_\_} = \_\_\_$$

- With $\alpha$ = .05, df=24, 2-tailed

  $t_{critical}$ = ______ (Table D.6; see next slide)
- Since ____ > ____ reject $H_0$



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| df | Critical Values of Student's t | | | | | |
|---|---|---|---|---|---|---|
| 1-tailed | 0.1 | **0.05** | **0.025** | 0.01 | 0.005 | 0.0005 |
| 2-tailed | 0.2 | **0.1** | **0.05** | 0.02 | 0.01 | 0.001 |
| 1 | 3.078 | **6.314** | **12.706** | 31.821 | 63.657 | 636.619 |
| 2 | 1.886 | **2.92** | **4.303** | 6.965 | 9.925 | 31.598 |
| 3 | 1.683 | **2.353** | **3.182** | 4.5415 | 5.841 | 12.941 |
| 4 | 1.533 | **2.132** | **2.776** | 3.747 | 4.604 | 8.61 |
| 5 | 1.476 | **2.015** | **2.571** | 3.365 | 4.032 | 6.859 |
| 6 | 1.44 | **1.943** | **2.447** | 3.143 | 3.707 | 5.959 |
| 7 | 1.415 | **1.895** | **2.365** | 2.998 | 3.499 | 5.405 |
| 8 | 1.397 | **1.86** | **2.306** | 2.896 | 3.355 | 5.041 |
| 9 | 1.383 | **1.833** | **2.262** | 2.821 | 3.25 | 4.781 |
| 10 | 1.372 | **1.812** | **2.228** | 2.764 | 3.169 | 4.587 |
| 11 | 1.363 | **1.796** | **2.201** | 2.718 | 3.106 | 4.437 |
| 12 | 1.356 | **1.782** | **2.179** | 2.681 | 3.055 | 4.318 |
| 13 | 1.35 | **1.771** | **2.16** | 2.65 | 3.012 | 4.221 |
| 14 | 1.345 | **1.761** | **2.145** | 2.624 | 2.977 | 4.14 |
| 15 | 1.341 | **1.753** | **2.131** | 2.602 | 2.947 | 4.073 |
| 16 | 1.337 | **1.746** | **2.12** | 2.583 | 2.921 | 4.015 |
| 17 | 1.333 | **1.74** | **2.11** | 2.567 | 2.898 | 3.965 |
| 18 | 1.33 | **1.734** | **2.101** | 2.552 | 2.878 | 3.922 |
| 19 | 1.328 | **1.729** | **2.093** | 2.539 | 2.861 | 4.883 |
| 20 | 1.325 | **1.725** | **2.086** | 2.528 | 2.845 | 3.85 |
| 21 | 1.323 | **1.721** | **2.08** | 2.518 | 2.831 | 3.819 |
| 22 | 1.321 | **1.717** | **2.074** | 2.508 | 2.819 | 3.792 |
| 23 | 1.319 | **1.714** | **2.069** | 2.5 | 2.807 | 3.767 |
| 24 | 1.318 | **1.711** | 2.064 | 2.492 | 2.797 | 3.745 |
| 25 | 1.316 | **1.708** | **2.06** | 2.485 | 2.787 | 3.725 |

## *T* DISTRIBUTION



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## USING *T* TO TEST $H_0 \rightarrow$ 1-TAILED$_{\alpha\,=\,.05}$

- $H_0$: $\mu \leq 100$
- $H_1$: $\mu > 100$ (One-tailed)
- The $t_{observed}$ value is the same → ______
- With $\alpha$ = .05, df=24, 1-tailed

  $t_{critical}$ = _____ (Table D.6; see next slide)
- Since ______ > ______ reject $H_0$

| df | Critical Values of Student's t | | | | | |
|---|---|---|---|---|---|---|
| 1-tailed | 0.1 | **0.05** | **0.025** | 0.01 | 0.005 | 0.0005 |
| 2-tailed | 0.2 | **0.1** | **0.05** | 0.02 | 0.01 | 0.001 |
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| 25 | 1.316 | **1.708** | **2.06** | 2.485 | 2.787 | 3.725 |

## *T* DISTRIBUTION



## CONCLUSIONS

- With $n$ = 25, $t_{observed}$(24) = _____
- Because _____ is larger than both _____ (1-tailed) and _____ (2-tailed) we reject $H_0$ under both 1- and 2-tailed hypotheses
- Conclude that taking IQPLUS leads to a higher IQ than normal.



## FACTORS AFFECTING…

- $t$ test
  - Difference between sample and population means
  - Magnitude of sample variance
  - Sample size
- Decision
  - Significance level α
  - One-tailed versus two-tailed test

## Size of the Effect

- We know that the difference is significant.
  - That doesn't mean that it is important.
- Population mean = 100, Sample mean = 106
- Difference is 6 words or roughly a 6% increase.
- Is this large?

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## Effect Size

- Later we develop this more in terms of standard deviations.
- For Example:
  - In our sample s = 7.78

$$Effect\ size = \frac{\bar{X} - \mu}{s} = \frac{106 - 100}{7.78} = \frac{6}{7.78} = .77$$

  - over 3/4 of a standard deviation

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## Confidence Intervals on Mean

- Sample mean is a point estimate
- We want interval estimate
- Given the sample mean we can calculate an interval that has a probability of containing the population mean
- This can be done if σ is known or not

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## Confidence Intervals on Mean

- If σ is known than the 95% CI is

$$CI_{.95} = \overline{X} \pm 1.96(\sigma_{\overline{X}})$$

- If σ is *not* known than the 95% CI is

$$CI_{.95} = \overline{X} \pm \left(t_{(2-tailed,\alpha=.05)} * s_{\overline{X}}\right)$$

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## For Our Data Assuming σ known

$$CI_{.95} = \overline{X} \pm \left(1.96 * \sigma_{\overline{X}}\right)$$

$$= 106 \pm (1.96 * \_\_\_\_)$$

$$= 106 \pm \_\_\_\_$$

$$= \_\_\_\_ \leq \mu \leq \_\_\_\_$$

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## For Our Data Assuming σ not known

$$CI_{.95} = \overline{X} \pm \left(t_{(2-tailed,\alpha=.05)} * s_{\overline{X}}\right)$$

$$= 106 \pm (\_\_\_\_ * \_\_\_\_)$$

$$= 106 \pm \_\_\_\_$$

$$= \_\_\_\_ \leq \mu \leq \_\_\_\_$$

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### CONFIDENCE INTERVAL

- Neither interval includes 100 - the population mean of IQ
- Consistent with result of *t* test.
- Confidence interval and effect size tell us about the magnitude of the effect.
- What else can we conclude from confidence interval?