

Math 140
Spring 10
Prof. Sethuraman
Sample Final
Time: Two hours

I will be looking for your work: show the formulas you are using, and the values that go into them. No credit if you just put down an answer to a question without any accompanying justification.

NOTE: Problems 3 through 6 on this sample are all on proportions. On the test, I could replace these with similar questions about means (with either sigma known or unknown) or variances as well.

1. In the Illinois Pick 3 lottery game, you pay 50 cents to select a sequence of three digits, such as 233. Meanwhile, the administrator of the game draws a particular sequence of three digits at random. If you have selected the same sequence as the administrator then you win and collect \$ 250 as prize money.
 - (a) How many different selections can you possibly make?
 - (b) What is the probability of your winning?
 - (c) If you win, what is your net profit?
 - (d) Find the expected value of your win/loss.
2. Assume that weights of newborn kittens are normally distributed with a mean of 3.5 ounces and a standard deviation of 0.26 ounces. If you were to declare that any kitten with a birthweight of 3.1 ounces or higher is “of safe birthweight” what percentage of the kitten population would be of safe birthweight?

3. A study intended to determine sleep patterns of college students involved a random sample of 313 students. 43% of them said they woke up more than once every night. What is a 95% confidence interval for the proportion of all students who wake up more than once each night?

4. Your city council suspects that not all households have complied with a recent law that requires all homes to install low-flow showerheads in their bathrooms. The council therefore needs to undertake a random inspection of a number of households to estimate the proportion of households that have not complied with the law. How many households must be inspected so that the council can be 95% certain of the proportion of non-compliant households, to within a margin of error $\pm 3\%$?

5. Kevin has heard that if you spin a penny by flicking it with your finger and letting it spin until it falls on the table, it is less likely land heads up than tails up. He spins a penny 40 times and gets heads 10 times. Determine whether this is statistically significant evidence that a spun penny is less likely to land heads up than tails up as follows:
- (a) Express the claim about the proportion p of spun pennies landing heads up as a mathematical inequality.
 - (b) Write down the logical opposite of your claim above.
 - (c) Write down the null hypothesis and alternative hypothesis.
 - (d) Compute the test statistic that you would use to assess the claim.
 - (e) Using a 95% confidence level, what is your conclusion about the claim? Write down your conclusion in terms of the likelihood of spun pennies landing heads less likely than tails (i.e., not merely in terms of rejecting or failing to reject the null hypothesis).

6. In a study of the connections between health and weight, researchers needed to determine if there was a variation between the proportions of overweight adults at the start and end of the five year period between 2004 and 2009. In a sample of a 1000 adults in 2004, 30% were found to be overweight, while in a sample of 1200 adults in 2009, 33% were found to be overweight. Determine if this is a significant increase from 2004 to 2009 in the proportion of those overweight at the 95% confidence level:

(a) Write down the null hypothesis and the alternative hypothesis.

(b) Is this a two-tailed or one-tailed test?

(c) Compute the test statistic.

(d) What is your conclusion about the claim?

7. You are given the following table that lists the mean concentration of CO₂ (in parts per million) in the atmosphere and the mean earth temperatures (in degrees Celsius) during different years.

CO ₂ (x)	314	317	320	326	331	339	346	354	361	369
Temp. (y)	13.9	14.0	13.9	14.1	14.0	14.3	14.1	14.5	14.5	14.4

You are told that the following values have been computed from the x and y values above:

Σx	Σy	Σxy	Σx^2	Σy^2
3377	141.7	47888.6	1143757	2008.39

Compute the (Pearson) correlation coefficient between CO₂ levels and temperature, using the data above and then use the correlation coefficient to determine if there is a linear correlation between CO₂ levels and temperature.

8. According to Benford's Law, the leading digits of checks occur with the following frequencies:

1	2	3	4	5	6	7	8	9
30.1%	17.6%	12.5%	9.7%	7.9%	6.7%	5.8%	5.1%	4.6%

An investigation of 784 checks issued by 7 suspect companies found that the leading digits in these 784 checks occurred with the following frequencies:

1	2	3	4	5	6	7	8	9
0	15	0	76	479	183	8	23	0

Use Benford's Law with a 0.01 significant level to test if there appears to be fraud behind these checks.