Essential Skills Chapter 3

1. **Simplifying the difference quotient** $\frac{f(x+h)-f(x)}{h}$ Section 3.1

   Example: For $f(x)=3-4x-4x^2$, find $\frac{f(x+h)-f(x)}{h}$ and simplify completely.

   Answer: $-4-8x-4h$

2. **Finding the domain of a function** Section 3.1

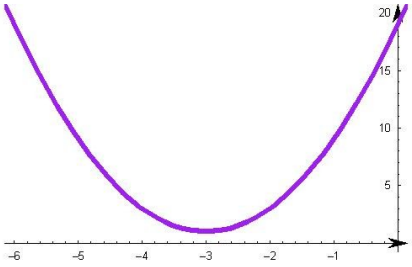
   Example: Find the domain of $f(x)=\frac{4}{\sqrt{x-9}}$.

   Answer: $(9,\infty)$

3. **Finding information from the graph of a function** Sections 3.2 and 3.3

   Example: The graph of a quadratic function *f* is given below. It's vertex is at $(-3,1)$.

   Use the graph to answer the following questions.



   a. On what interval is the function increasing? Answer: $[-3,\infty)$

   b. On what interval is the function decreasing? Answer: $(-\infty,-3]$

   c. What is the domain of the function? Answer: $(-\infty,\infty)$

   d. What is the range of the function? Answer: $[1,\infty)$

   e. Which does this function have, a maximum or minimum value? What is it? Answer: A minimum value; 1

   f. Which of the following could be the formula for $f(x)$? Answer: iv

      i. $f(x)=(x+3)^2+1$ ii. $f(x)=2(x-1)^2+3$ iii. $f(x)=2(x-3)^2+1$

      iv. $f(x)=2(x+3)^2+1$

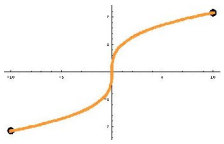
4. **Finding the average rate of change of a function** Section 3.3

   Example: Find the average rate of change of the function $f(x)=\sqrt[3]{1-x}$ on the invertval $[-7,9]$.

   Answer: $-\frac{1}{4}$

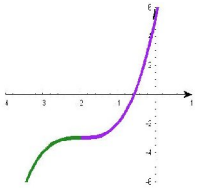
5. **Sketching graphs of basic functions** Section 3.4

Example: Sketch the graph of $f(x) = \sqrt[3]{x}$.

Answer: 

6. **Sketching graphs of basic functions using transformations** Section 3.5

Example: Sketch the graph of $f(x) = (x+2)^3 - 3$

Answer: 

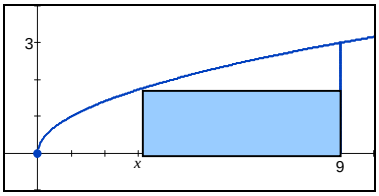
7. **Constructing functions for modeling** Section 3.6

Example: Consider the region in the plane bounded by $y = \sqrt{x}$, $x = 9$ and the x-axis.

Each value of x, $0 \le x \le 9$, corresponds to an inscribed rectangle whose right side lies

along the line $x = 9$ (see figure). Write a function that expresses the area of the inscribed

rectangle as a function of x.

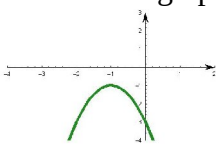


Answer: $f(x) = \sqrt{x}(9-x)$

Essential Skills Chapter 4

1. **Graphing quadratic functions** Section 4.1

   Example: Sketch the graph of $f(x) = -2x^2 - 4x - 3$. Label the vertex and *y*-intercept.

   Answer: 

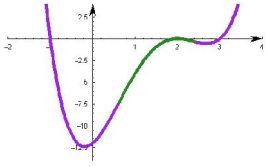
2. **Finding optimal values of quadratic models** Section 4.1

   Example: Paradise Travel Agency's monthly profit *P* (in thousands of dollars) depends on the amount of money *x* (in thousands of dollars) spent on advertising per month according to the rule $P(x) = 7 - 2x(x-4)$. What is Paradise's maximum monthly profit?

   Answer: $15,000

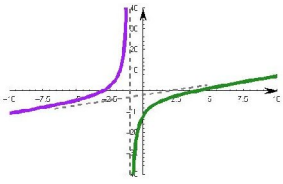
3. **Graphing polynomial functions** Section 4.2

   Example: Sketch the graph of $f(x) = (x-2)^2(x-3)(x+1)$.

   Answer: 

4. **Graphing rational functions** Sections 4.3 and 4.4

   Example: Sketch the graph of $R(x) = \dfrac{x^2 - x - 12}{x+1}$.

   Answer: 

5. **Solving rational inequalities** Section 4.5

   Example: Solve. $\dfrac{x}{x+2} \le \dfrac{1}{x}$

   Answer: $x \in (-2,-1] \cup (0,2]$

6. **Finding zeros of polynomials** Sections 4.6 and 4.7

   Example: Find all the zeros of $P(x) = 2x^3 - 5x^2 + 6x - 2$

   Answer: $\frac{1}{2}, 1+i, 1-i$

Essential Skills Chapter 5

1. **Finding composite functions and their domains** Section 5.1

   Example: For $f(x)=\dfrac{1}{x+3}$ and $g(x)=\dfrac{1}{x-2}$, find $(f\circ g)(x)$ and it's domain.

   Answer: $(f\circ g)(x)=\dfrac{x-2}{3x-5}$, domain = $\left\{x\middle| x\neq \frac{5}{3},2\right\}$

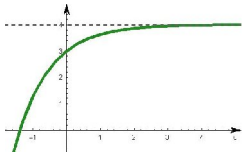
5. **Finding inverse functions** Section 5.2

   Example: For $f(x)=\dfrac{1}{3x-2}$, find $f^{-1}(x)$.

   Answer: $f^{-1}(x)=\dfrac{1+2x}{3x}$

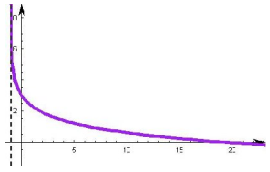
6. **Graphing exponential functions** Section 5.3

   Example: Sketch the graph of $f(x)=4-e^{-x}$.

   Answer: 

7. **Graphing logarithmic functions** Section 5.4

   Example: Sketch the graph of $f(x)=3-\log(x+1)$.

   Answer: 

5. **Simplifying expressions involving logarithms** Section 5.5

   Example: Write as a single logarithm. $20\log_2\sqrt[4]{x}+\log_2(4x^3)-\log_2 4$

   Answer: $\log_2(x^8)$

6. **Solving logarithmic equations** Section 5.6

   Example: Solve. $\log_{15} x+\log_{15}(x-2)=1$

   Answer: $x=5$

7. **Solving exponential equations** Section 5.6

   Example: Solve. $2^{x+3}=5^x$

   Answer: $x=\dfrac{-3\ln 2}{\ln 2-\ln 5}$

8. **Modeling using exponential functions** Section 5.8

Example: A population of bacteria obeys the law of uninhibited growth. If 600 bacteria are present initially and there are 800 after one hour,

a. Express the population P as a function of time t.
b. How long will it be until the population doubles? (Write an exact answer.)

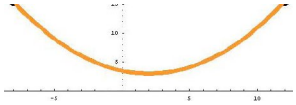
Answer: a. $P(t) = 600e^{t\ln(\frac{4}{3})}$ b. $\dfrac{\ln 2}{\ln(\frac{4}{3})}$ hours

Essential Skills Chapter 6

1. **Finding the center and radius of a circle** Section 2.3
   Example: Find the center and radius of the circle with equation $x^2 + y^2 - 6x + 10y + 25 = 0$.

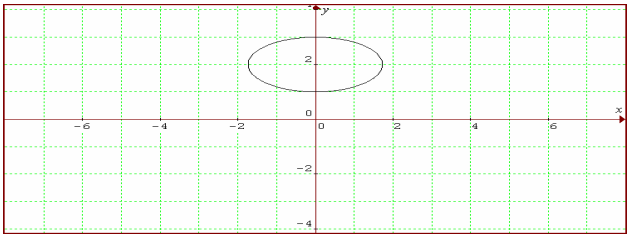
   Answer: center is $(3,-5)$, radius is 3

2. **Graphing parabolas** Section 6.2
   Example: For the parabola defined by the equation $x^2 - 4x = 8y - 28$, determine the vertex, focus, and directrix and sketch the graph.

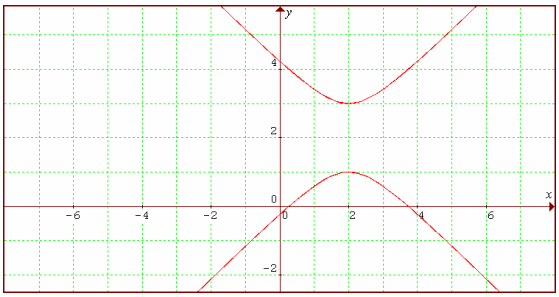
   

   Answer: vertex is $(2,3)$, focus is $(2,5)$, directrix is $y = 1$

3. **Graphing ellipses** Section 6.3
   Example: For the ellipse defined by the equation $x^2 + 3y^2 - 12y + 9 = 0$, determine the center, vertices, and foci and sketch the graph.
   Answer: center is $(0,2)$, vertices are $(-\sqrt{3},2)$ and $(\sqrt{3},2)$, foci are $(-\sqrt{2},2)$ and $(\sqrt{2},2)$

4. **Graphing Hyperbolas** Section 6.4
   Example: For the hyperbola defined by the equation $y^2 - x^2 + 4x - 4y - 1 = 0$, determine the center, vertices, foci, transverse axis, asymptotes and sketch the graph..
   Answer: center is $(2,2)$, vertices are $(2,1)$ and $(2,3)$,
   foci are $(2,2-\sqrt{2})$ and $(2,2+\sqrt{2})$, transverse axis is $x = 2$,
   asymptotes are $y - 2 = x - 2$ and $y - 2 = -(x - 2)$

Essential Skills Chapter 7

1. **Solving systems of linear equations** Section 7.1

   Example: Solve. $\begin{cases} .5x + .3y = 2.7 \\ .7x - .2y = 1.3 \end{cases}$

   Answer: $x = 3,\ y = 4$

8. **Using matrices to solve systems of linear equations** Section 7.2

   Example: Solve using matrices. $\begin{cases} x + y - z - w = 6 \\ 2x + z - 3w = 8 \\ x - y + 4w = -10 \\ 3x + 5y - z - w = 20 \end{cases}$

   Answer: $x = 1,\ y = 3,\ z = 0,\ w = -2$

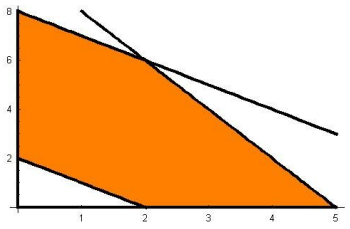
9. **Solving systems of nonlinear equations** Section 7.6

   Example: Solve. $\begin{cases} x^2 + y^2 = 100 \\ 3x - y = 10 \end{cases}$

   Answer: $(0,-10)$ and $(6,8)$

10. **Graphing systems of linear inequalities** Section 7.7

    Example: Graph. $\begin{cases} x \geq 0 \\ y \geq 0 \\ x + y \geq 2 \\ x + y \leq 8 \\ 2x + y \leq 10 \end{cases}$ Answer:



11. **Problem solving with Linear Programming** Section 7.8

Process	**Hours, model A**	**Hours, model B**
Assembling	2	2.5

Painting	4	1
Packaging	1	0.75

Example: A manufacturer produces two models of bicycles. The times (in hours) required for assembling, painting, and packaging each model are shown in the table.

The total times available for assembling, painting, and packaging are 4000 hours, 4800 hours, and 1500 hours, respectively. The profits per unit are $45 for model A and $50 for model B. How many of each type should be produced to maximize profit? What is the maximum profit?

Answer: 750 units of model A, 1000 units of model B; maximum profit: $83,750

Essential Skills Chapter 8

1. **Using Mathematical Induction** Section 8.4

   Example: Prove that $n < 2^n$ for all positive integers *n*.

   Answer: Proof: For n $n = 1$, the statement is true, because $1 < 2^1$. Assuming that $k < 2^k$, we need to show that $k + 1 < 2^{k+1}$. For $n = k$, we have $2^{k+1} = 2(2^k) > 2k$ (by assumption). Because $2k = k + k > k + 1$ for all $k > 1$, it follows that $2^{k+1} > 2k > k + 1$, that is, $k + 1 < 2^{k+1}$. Therefore, $n < 2^n$ for all integers $n \geq 1$.