Essential Skills Chapter 3

1. **Simplifying the difference quotient** $\dfrac{f(x+h)-f(x)}{h}$ Section 3.1

   Example: For $f(x)=3-4x-4x^2$, find $\dfrac{f(x+h)-f(x)}{h}$ and simplify completely.

   Answer: $-4-8x-4h$

2. **Finding the domain of a function** Section 3.1

   Example: Find the domain of $f(x)=\dfrac{4}{\sqrt{x-9}}$.

   Answer: $(9,\infty)$

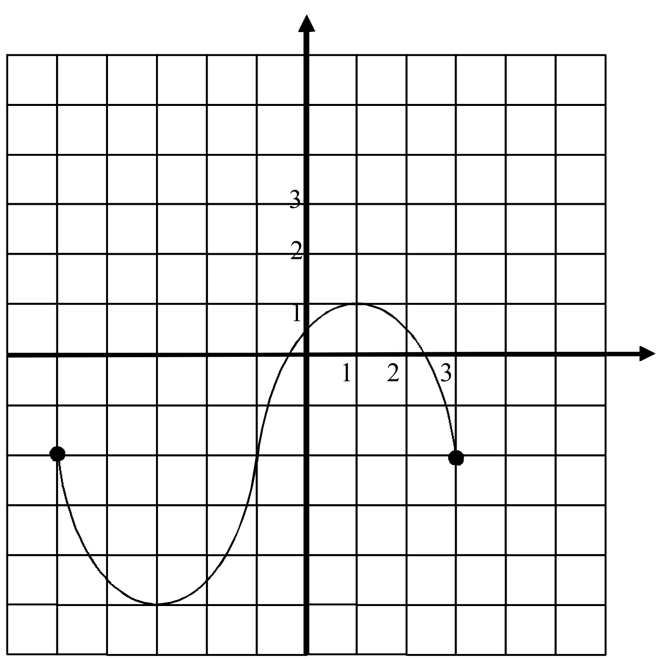
3. **Using functions as models to make predictions and draw conclusions** Section 3.1

   Example: If a rock falls from a height of 20 meters on the planet Jupiter, its height H (in meters) after x seconds is approximately $H(x)=20-13x^2$.

   a. What is the height of the rock after 1 second? Answer: 7 meters
   b. When is the height of the rock 10 meters? Answer: After approximately .88 seconds
   c. When does the rock strike the ground? Answer: After approximately 1.24 seconds

4. **Finding information from the graph of a function** Sections 3.2 and 3.3

   Example: Use the graph of the function f below to answer the following:



   a. What is $f(3)$? Answer: -2
   b. What is $f(2)$ approximately? Answer: $\frac{1}{2}$

c. What is the average rate of change of f on the interval $[-1,1]$? Answer: $\frac{3}{2}$

d. What is a local minimum value of f? Answer: -5

e. On what interval(s) is f increasing? Answer: $[-3,1]$

f. On what interval(s) is f decreasing? Answer: $[-5,-3]\cup[1,3]$

g. What is the domain of f? Answer: $[-5,3]$

h. What is the range of f? Answer: $[-5,1]$

i. For what values of x is $f(x)<-2$? Answer: $\{x \mid -5<x<-1\}$

j. For what value of x is $f(x)=-5$? Answer: -3

5. **Identifying the relationship between a function and its graph** Section 3.2

Example: Let $f(x)=\dfrac{x^2+2}{x+4}$

a. Is the point $\left(1,\frac{3}{5}\right)$ on the graph of f? Answer: yes

b. If $x=0$, what is $f(x)$? What corresponding point is on the graph of f? Answers: $\frac{1}{2}$, $\left(0,\frac{1}{2}\right)$

c. If $f(x)=\frac{1}{2}$, what corresponding points are on the graph of f?

d. Find the x and y intercepts of the graph of f. Answers: x-intercept: none, y-intercept: $\left(0,\frac{1}{2}\right)$

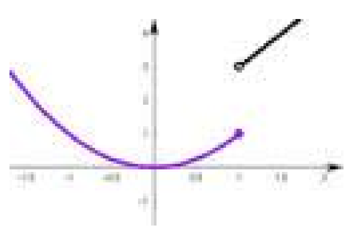
6. **Finding the average rate of change of a function** Section 3.3

Example: Find the average rate of change of the function $f(x)=\sqrt[3]{1-x}$ on the interval $[-7,9]$.

Answer: $-\dfrac{1}{4}$

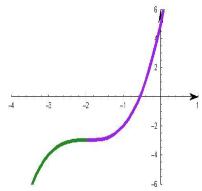
7. **Sketching graphs of basic functions** Section 3.4

Example: Sketch the graph of $f(x)=\begin{cases} x^2 & \text{if } x\le 1 \\ 2x+1 & \text{if } x>1 \end{cases}$.

Answer: 

8. **Sketching graphs of basic functions using transformations** Section 3.5

Example: Sketch the graph of $f(x)=(x+2)^3-3$

Answer: 

9. **Constructing functions for modeling** Section 3.6

Example: A rectangle with width x has a perimeter of 24 inches.

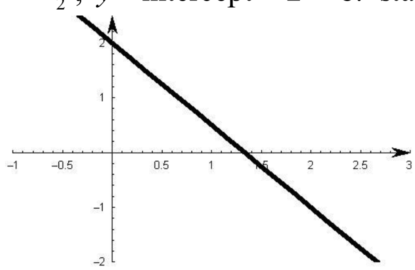
a. Express the length of the rectangle l as a function of x. Answer: $l(x) = 12 - x$

b. Find the domain of l. Express your answer in interval notation. Answer: $(0,12)$

c. Express the area of the rectangle A as a function of x. Answer: $A(x) = 12x - x^2$

Essential Skills Chapter 4

1. **Identifying properties of linear functions** Section 4.1
   Example: If $f(x) = -\frac{3}{2}x + 2$,
   a. Determine the slope and y-intercept of f.
   b. Use the slope and y-intercept to graph f.
   c. Determine the average rate of change of f on the interval $\left[.3, \frac{4}{9}\right]$
   d. Determine whether f is increasing, decreasing, or constant.
   Answers: a. slope $= -\frac{3}{2}$, $y-$intercept $= 2$ b. start at $(0,2)$, then go down three and right two



   to $(2,-1)$
   c. $-\frac{3}{2}$ d. decreasing

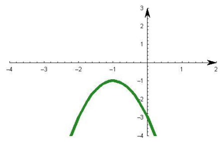
2. **Using linear functions as models** Section 4.1
   Example: In 2002, major league baseball signed a labor agreement with the players. In this agreement, any team whose payroll exceeds \$128 million starting in 2005 will have to pay a luxury tax of 22.5% (for first-time offenses). The linear function $T(p) = 0.225(p-128)$ describes the luxury tax T of a team whose payroll is p (in millions of dollars).
   a. What is the implied domain of this function?
   b. What is the luxury tax for a team whose payroll is is \$160 million?
   c. What is the payroll of a team that pays a luxury tax of \$11.7 million?

   Answers: a. $\{p \mid p \geq 128\}$ b. \$7.2 million c. \$180 million

3. **Identifying properties of quadratic functions** Section 4.3
   Example: Let $f(x) = -2x^2 - 4x - 3$.
   a. Express f in the form $f(x) = a(x-h)^2 + k$
   b. Find the vertex of the graph of f.
   c. Find the axis of symmetry of the graph of f.
   d. Find the x and y intercepts of the graph of f.
   e. Where is f increasing and decreasing?
   f. Find the domain and range of f.
   g. Does f have a maximum or minimum value? What is the maximum/minimum value?
   h. Sketch the graph of f.

Answers: a. $f(x) = -2(x+1)^2 - 1$ b. $(-1,-1)$ c. $x = -1$ d. x-intercepts: none, y-intercept: $(0,-3)$

e. increasing on $(-\infty,-1]$, decreasing on $[-1,\infty)$ f. domain: $(-\infty,\infty)$, range: $(-\infty,-1]$ g. maximum; -1

h. 

2. 4. **Finding optimal values of quadratic models** Section 4.3

Example: Paradise Travel Agency's monthly profit P (in thousands of dollars) depends on the amount of money x (in thousands of dollars) spent on advertising per month according to the rule $P(x) = 7 - 2x(x-4)$. What is Paradise's maximum monthly profit?

Answer: \$15,000

5. **Constructing and using quadratic models** Section 4.4

Example: A farmer with 2000 meters of fencing wants to enclose a rectangular plot that borders on a straight highway. If the farmer does not fence the side along the highway,

a. Express the area, A, of the plot as a function of x (where x is the side of the plot perpendicular to the highway).

b. What is the largest area that can be enclosed?

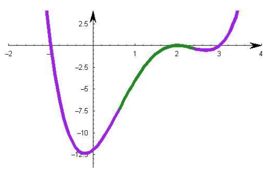
Answers: a. $A(x) = 2000x - 2x^2$ b. 500,000 square meters

Essential Skills Chapter 5

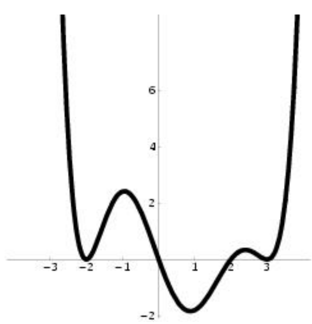
1. **Graphing polynomial functions** Section 5.1
   Example: Let $f(x)=(x-2)^2(x-3)(x+1)$.

   a. Find the x and y intercepts of the graph of f.
   b. Determine the end behavior of f.
   c. Sketch the graph of f.

   Answers: a. x-intercepts: $(2,0)$, $(3,0)$, $(-1,0)$, y-intercept: $(0,-12)$
   b. as $x\to\infty$, $y\to\infty$; as $x\to-\infty$, $y\to\infty$

   c. 

2. **Identifying properties of polynomial functions** Section 5.1
   Example: Use the graph of a polynomial function $P(x)$ to answer the following questions.



   a. What is the minimum degree of $P(x)$?
   b. What is the sign of the leading coefficient of $P(x)$?
   c. Is the degree of $P(x)$ odd or even?
   d. Which of the following is most likely to be the formula for $P(x)$? (circle one)

      i) $P(x)=x(x+2)(x-2)(x-3)$
      ii) $P(x)=x(x+2)^2(x-2)(x-3)$
      iii) $P(x)=x(x+2)^2(x-2)(x-3)^2$
      iv) $P(x)=x(x+2)(x-2)^2(x+3)^2$

   Answers: a. 6 b. positive c. even d. iii

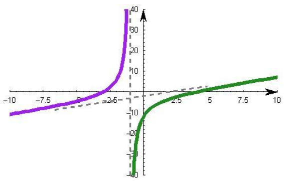
3. **Graphing rational functions** Sections 5.2 and 5.3

<u>Example</u>: Let $R(x)=\dfrac{x^2-x-12}{x+1}$.

a. Find the x and y intercepts of the graph of R.
b. Find the equations of all asymptotes.
c. Sketch the graph of R.

Answers: a. x-intercepts: $(4,0)$, $(-3,0)$, y-intercept: $(0,-12)$

b. vertical asymptote: $x=-1$, horizontal asymptote: none, oblique asymptote: $y=x-2$

c. 

4. **Solving polynomial and rational inequalities** Section 5.4

<u>Example</u>: Solve. $\dfrac{x}{x+2}\le\dfrac{1}{x}$

Answer: $x\in(-2,-1]\cup(0,2]$

5. **Finding zeros of polynomials** Sections 5.5 and 5.6

<u>Example</u>: Find all the zeros of $P(x)=2x^3-5x^2+6x-2$

Answer: $\frac{1}{2},1+i,1-i$

6. **Writing the equation of a polynomial using its zeros Section** 5.6

Example: Write the equation of the fourth degree polynomial $P(x)$ having $1-i$ a zero, -2 a zero of multiplicity 2, and a leading coefficient of 3.

Answer: $P(x)=3x^4+6x^3-6x^2+24$

Essential Skills Chapter 6

1. **Finding composite functions and their domains** Section 6.1

   Example: For $f(x) = \dfrac{1}{x+3}$ and $g(x) = \dfrac{1}{x-2}$, find $(f \circ g)(x)$ and it's domain.

   Answer: $(f \circ g)(x) = \dfrac{x-2}{3x-5}$, domain = $\left\{x \middle| x \neq \frac{5}{3}, 2\right\}$

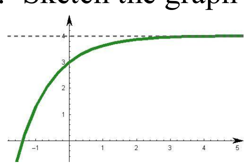
2. **Finding inverse functions** Section 6.2

   Example: For $f(x) = \dfrac{1}{3x-2}$, find $f^{-1}(x)$.

   Answer: $f^{-1}(x) = \dfrac{1+2x}{3x}$

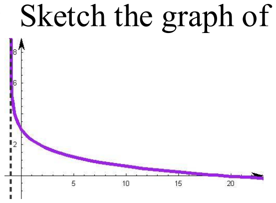
3. **Graphing exponential functions** Section 6.3

   Example: Sketch the graph of $f(x) = 4 - e^{-x}$.

   Answer: 

4. **Graphing logarithmic functions** Section 6.4

   Example: Sketch the graph of $f(x) = 3 - \log(x+1)$.

   Answer: 

5. **Simplifying expressions involving logarithms** Section 6.5

   Example: Write as a single logarithm. $20\log_2 \sqrt[4]{x} + \log_2(4x^3) - \log_2 4$

   Answer: $\log_2(x^8)$

6. **Solving logarithmic equations** Section 6.6

   Example: Solve. $\log_{15} x + \log_{15}(x-2) = 1$

   Answer: $x = 5$

7. **Solving exponential equations** Section 6.6

   Example: Solve. $2^{x+3} = 5^x$

Answer: $x = \dfrac{-3\ln 2}{\ln 2 - \ln 5}$

8. **Modeling using exponential functions** Section 6.8

Example: A population of bacteria obeys the law of uninhibited growth. If 600 bacteria are present initially and there are 800 after one hour,

a. Express the population P as a function of time t.
b. How long will it be until the population doubles? (Write an exact answer.)

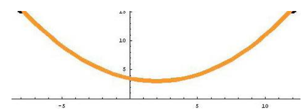
Answer: a. $P(t) = 600e^{t\ln(\frac{4}{3})}$ b. $\dfrac{\ln 2}{\ln(\frac{4}{3})}$ hours

Essential Skills Chapter 7

1. **Finding the center and radius of a circle** Section 2.4
   Example: Find the center and radius of the circle with equation $x^2 + y^2 - 6x + 10y + 25 = 0$.

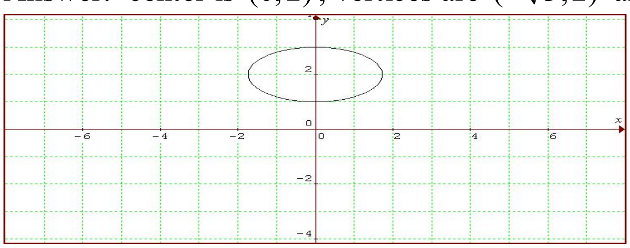
   Answer: center is $(3,-5)$, radius is 3

2. **Graphing parabolas** Section 7.2
   Example: For the parabola defined by the equation $x^2 - 4x = 8y - 28$, determine the vertex, focus, and directrix and sketch the graph.

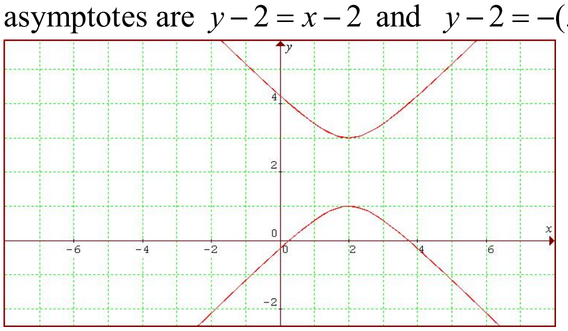
   

   Answer: vertex is $(2,3)$, focus is $(2,5)$, directrix is $y = 1$

3. **Graphing ellipses** Section 7.3
   Example: For the ellipse defined by the equation $x^2 + 3y^2 - 12y + 9 = 0$, determine the center, vertices, and foci and sketch the graph.
   Answer: center is $(0,2)$, vertices are $(-\sqrt{3},2)$ and $(\sqrt{3},2)$, foci are $(-\sqrt{2},2)$ and $(\sqrt{2},2)$

4. **Graphing hyperbolas** Section 7.4
   Example: For the hyperbola defined by the equation $y^2 - x^2 + 4x - 4y - 1 = 0$, determine the center, vertices, foci, transverse axis, asymptotes and sketch the graph..
   Answer: center is $(2,2)$, vertices are $(2,1)$ and $(2,3)$,
   foci are $(2, 2-\sqrt{2})$ and $(2, 2+\sqrt{2})$, transverse axis is $x = 2$,
   asymptotes are $y - 2 = x - 2$ and $y - 2 = -(x - 2)$

5. **Using the equations of conics to solve applied problems** Sections 7.2, 7.3, 7.4
Example: A satellite dish is shaped like a paraboloid of revolution. The signals that emanate from a satellite strike the surface of the dish and are reflected to a single point, where the receiver is to be located. If the dish is 10 feet across at its opening and 6 feet deep at its center, at what position should the receiver be placed?
Answer: It should be placed 1 foot and $\frac{1}{2}$ inch from its base along its axis of symmetry.

Essential Skills Chapter 8

1. **Solving systems of linear equations** Section 8.1

   Example: Solve. $\begin{cases} .5x + .3y = 2.7 \\ .7x - .2y = 1.3 \end{cases}$

   Answer: $x = 3$, $y = 4$

2. **Solving systems of nonlinear equations** Section 8.6

   Example: Solve. $\begin{cases} x^2 + y^2 = 100 \\ 3x - y = 10 \end{cases}$

   Answer: $(0, -10)$ and $(6, 8)$

3. **Using systems of equations to solve applied problems** Section 8.1

   Example: Find real numbers *a*, *b*, and *c* so that the function $y = ax^2 + bx + c$ contains the points $(-1, 6)$, $(2, 8)$, and $(0, 4)$.

   Answer: $y = \frac{4}{3}x^2 - \frac{2}{3}x + 4$