

Hypothesis Testing

Steps for a hypothesis test:

1. State the claim H_0 and the alternative, H_a
2. Choose a significance level or use the given one.
3. Draw the sampling distribution based on the assumption that H_0 is true, and shade the area of interest.
4. Check conditions. Calculate the test statistic.
5. Find the p-value.
6. If the p-value is less than the significance level, α , reject the null hypothesis. (There is enough evidence to reject the claim.)

If the p-value is greater than the significance level, α , do not reject the null hypothesis. (There is not enough evidence to reject the claim.)

(You can use the p-value to make a statement about the strength of the evidence against H_0 without using α , e.g., $p > .10$ implies “little or no evidence against H_0 ”, $.05 < p \leq .10$ implies “some evidence against H_0 ”, $.01 < p \leq .05$ implies “good evidence against H_0 ”, $.001 < p \leq .01$ implies “strong evidence against H_0 ”, $p \leq .001$ implies “very strong evidence against H_0 ”.)

7. Write a statement to interpret the decision in the context of the original claim.

Test statistics: (Step 3)

Hypothesis testing for a mean (σ is known, and the variable is normally distributed in the population or $n > 30$)

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

(TI-83: STAT \rightarrow TESTS \rightarrow 1:Z-Test)

Hypothesis testing for a mean (σ is unknown, and the variable is normally distributed in the population or $n > 30$)

$$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

(TI-83: STAT \rightarrow TESTS \rightarrow 2:T-Test)

Hypothesis testing for a proportion (when $np_0 \geq 10$ and $n(1 - p_0) \geq 10$)

(TI-83: STAT \rightarrow TESTS \rightarrow 5:1-PropZTest)

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

Hypothesis Testing for Proportion

1. Write the null and alternative hypotheses you would use to test each of the following situations:

a. Is a coin fair?

$$H_0: p = \frac{1}{2}$$

$$H_a: p \neq \frac{1}{2}$$

b. Only 34% of people who try to quit smoking succeed. A company claims that using their chewing gum can help people quit.

$$H_0: p = 0.34$$

$$H_a: p > 0.34$$

c. In the 1950s only about 40% of high school graduates went on to college. Has the percentage changed?

$$H_0: p = 0.40$$

$$H_a: p \neq 0.40$$

d. A large city's DMV claimed that 80% of candidates pass driving tests, but a newspaper reporter claims that this rate is lower than the DMV's reported value

$$H_0: p = 0.80$$

$$H_a: p < 0.80$$

2. In a 1993 Gallup poll, 47% of the respondents agreed with the statement "*God created human beings pretty much in their present form at one time within the last 10,000 years or so.*" When Gallup asked the same question in 2001, only 45% of those responded agreed. A hypothesis test had been carried out for the difference in public opinion. Is it reasonable to conclude that there was a change in public opinion given that the p-value is 0.37? Explain.

No, it's not reasonable to conclude that there was a change in public opinion since the p-value is big, greater than 5%, even greater than 10%.

3. Your friend claims that he has a fair die, but wants to bet on the number 6 with you. You suspect that his die is loaded, and you decide to roll the die 200 times. The appropriate hypothesis testing provides a p-value of 0.04. Which conclusion is correct at 5% significance level?

a. There is a 4% chance that the die is fair.

b. There is a 4% chance that the die is loaded.

c. There's a 4% chance that a fair die could produce results we observed just by chance, so it's reasonable to conclude that the die is fair.

d. There's a 4% chance that a fair die could produce results we observed just by chance, so it's reasonable to conclude that the die is loaded.

4. The Pew Research Center claims that in 2007 34% of Internet users in the United States have logged onto the internet using a wireless network in their home, at their workplace, or place else. A skeptical citizen collected data from a random sample of 378 adults because he believed that more than 34% of Internet users use a wireless network. 149 of them said they have logged onto the

internet using a wireless network. At $\alpha = 10\%$, is there enough evidence to reject the researcher's claim? (www.pewinternet.org)

We did this in class.

5. The Pew Research Center also claims that 87% of Internet users who ever use maps or get driving directions do this online. A map publishing company believes that not that many use maps and driving directions online. In a random sample of 250 adults, 83% say they go online to get maps or driving directions. At $\alpha = 5\%$, is there enough evidence to reject the researcher's claim? At $\alpha = 10\%$ level?

$$H_0: p = 0.87$$

$$H_a: p < 0.87$$

$$\text{Conditions: SRS, } n\hat{p} = 250(0.83) = 207.5 > 10 \quad n(1 - \hat{p}) = 250(1 - 0.83) = 42.5 > 10$$

Test statistics and p-value: (Using STAT \rightarrow TESTS \rightarrow 5: 1-PropZTest with $p_0 = 0.87$, $x = 208$, and $n = 250$. (x is $250(0.83) = 207.5$ but round it up)

$$z = -1.787$$

$$p\text{-value} = 0.037$$

Both at the 5% and 10% significance level we have enough evidence to reject the null hypothesis since the p-value is less than 5% (and less than 10%). That is, we can conclude that less than 87% of Internet users who ever use maps or get driving directions do this online.

6. In the 1980s it was generally believed that congenital abnormalities affected about 5.2% of the nation's children. Some people believe that the increase in the number of chemicals in the environment has led to an increase in the incidence of abnormalities. A recent study examined 384 children and found that 28 of them showed signs of abnormality. At the 5% level, is this strong evidence that the risk has increased? At the 1% level?

$$H_0: p = 0.052$$

$$H_a: p > 0.052$$

$$\hat{p} = \frac{28}{384} = 0.073$$

$$\text{Conditions: SRS, } n\hat{p} = 384(0.073) = 28 > 10 \quad n(1 - \hat{p}) = 384(1 - 0.073) = 366 > 10$$

Test statistics and p-value: (Using STAT \rightarrow TESTS \rightarrow 5: 1-PropZTest with $p_0 = 0.052$, $x = 28$, and $n = 384$.)

$$z = 1.846$$

$$p\text{-value} = 0.0324$$

At the 5% significance level we have good (not strong) evidence to reject the null hypothesis since the p-value is less than 5%. That is, we can conclude that more than 5.2% of the nation's children have congenital abnormalities.

At the 1% significance level we don't have enough evidence to reject the null hypotheses since the p-value is greater than 1%. That is, we can't reject the claim that 5.2% of the nation's children have congenital abnormalities.

7. A research center estimates that 40% of U.S. adults eat breakfast every day. In a random sample of 300 U.S. adults, 135 say they eat breakfast every day. At the 10% level, is there enough evidence to reject the researcher's claim? At the 5% level?

$$H_0: p = 0.40$$

$$H_a: p \neq 0.40$$

$$\hat{p} = \frac{135}{300} = 0.45$$

$$\text{Conditions: SRS, } n\hat{p} = 300(0.45) = 135 > 10$$

$$n(1 - \hat{p}) = 300(1 - 0.45) = 165 > 10$$

Test statistics and p-value: (Using STAT → TESTS → 5: 1-PropZTest with $p_0 = 0.40$, $x = 135$, and $n = 300$.)

$$z = 1.768$$

$$p\text{-value} = 0.077$$

At the 10% significance level we have enough evidence to reject the null hypothesis since the p-value is less than 10%. That is, we can conclude that the proportion of U.S. adults who eat breakfast every day is not 40%.

At the 5% level we don't have enough evidence to reject the null hypothesis since the p-value is greater than 5%. That is we cannot reject the claim that 40% of U.S. adults eat breakfast every day.

Hypothesis Testing for Mean

1. In 1960, census results indicated that the age at which American men first married had a mean of 23.3 years. It is widely suspected that young people today are waiting longer to get married. We plan to test our hypothesis by selecting a random sample of 20 men who married for the first time last year. The men in our sample married at an average age of 24.2 years, with standard deviation of 5.1 years. Find the errors in a student's attempt to test an appropriate hypothesis. How many mistakes can you find?

$$H_0: \bar{x} \geq 23.3$$

$$H_a: \bar{x} < 23.3$$

$$z = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{23.3 - 24.2}{\frac{5.1}{\sqrt{20}}} = -0.789$$

We never ever ever ever test a sample statistic. We KNOW everything about the sample, so we don't need to test it. Thus they should be μ , not \bar{x} . Also, we suspect that younger people are waiting LONGER to get married, so it should be ">" in the alternative hypothesis.

In the formula \bar{x} is 24.2 and not 23.3. So the numerator should be switched: $24.2 - 23.3$. The $z = 0.789$.

$$p\text{-value} = 0.2151$$

Conclusion: there is more than 21% chance that the mean age men first get married is 23.3. Thus, there is strong evidence that the average year men first get married has not increased since 1960.

The conclusion is incorrect. Since the p-value is larger (greater than even 10%) we don't have evidence to reject the null hypothesis. That is, we don't have evidence to reject the claim that the average age men get first married is 23.3 years.

Did you know Santa once took a statistics class?

He had trouble remembering which hypothesis should have the equal sign so he would keep repeating: the null hypothesis, the null hypothesis, the null hypothesis. In fact to this day you can hear him say **Ho, Ho, Ho!**

2. Humerus bones from members of the same species of animal tend to have approximately the same length-to-width ratio, which are usually different from that of other species. When fossils of humerus bones are discovered, therefore, archeologists can often determine the species of animal by measuring the length-to-width ratios of the bones. Suppose that it is known that species A exhibits a mean ratio of 8.5, and suppose 41 fossils of humerus bones were unearthed at an archeological site in East Africa, where species A is believed to have lived, and the length-to-width ratios of the bones were calculated. The first three and the last three of them are shown below:

10.73 8.89 9.07 ... 9.17 12.00 9.38

In order to decide whether the fossils found were indeed from species A, the archeologists want to test at the 5% level whether the population mean ratio for all bones of the species that lived at this site is 8.5 or differs from 8.5.

a. State the hypotheses.

Did this in class.

b. Use the Minitab output to test the claims. Use both outputs (T-Test of the Mean, and T Confidence Intervals) in your decision. Write your conclusions in context.

Did this in class.

T-Test of the Mean

Test of mu = 8.500 vs mu not = 8.500

Variable	N	Mean	StDev	SE Mean	T	P
RATIO	41	9.258	1.204	0.188	4.03	0.0002

T Confidence Intervals

Variable	N	Mean	StDev	SE Mean	95.0 % CI
RATIO	41	9.258	1.204	0.188	(8.878, 9.637)

3. Industrial pollution often makes water supplies acidic (pH < 7) or basic (pH > 7). An industrial company claims that the mean pH level of the water in a nearby river is 7, so it is neutral. You, as an environmentalist, randomly select 19 water samples and measure the pH of each. The

sample mean and standard deviation are 6.9 and 0.24, respectively. Assume the population of pH levels in all possible water samples is normally distributed.

a. Is there enough evidence to reject the company's claim at the 5% significance level? At the 10% level?

$$H_0: \mu = 7 \qquad H_a: \mu \neq 7$$

Conditions: SRS, normal distribution

Test statistics and p-value: (Using STAT → TESTS → 2: TTest (since sigma is unknown) with $\mu_0 = 7$, $\bar{x} = 6.9$, $S_x = 0.24$ and $n = 19$.)

$$t = -1.82 \qquad p\text{-value} = 0.086$$

At the 5% significance level we do not have enough evidence to reject the null hypothesis since the p-value is greater than 5%. That is, we cannot reject the claim that the pH level of the water in the nearby river is 7.

At the 10% significance level we have enough evidence to reject the null hypotheses since the p-value is less than 10%. That is, we can conclude that the pH level of the water in the nearby river is not 7.

b. Can you reach the same conclusion using the appropriate confidence interval for the mean pH level?

Yes, because it's a two sided test (we have a "≠" in the alternative hypothesis). Using STAT → TESTS → 8: TInterval, the 95% CI is (6.7843, 7.0157). Since the claimed value, 7, is in the confidence interval, we can't reject the claim at the 5% level.

(the confidence level and the significance level must add up to 100%, so that's why I said "at the 5% level").

The 90% is (6.8045, 6.9955). Since the claimed value, 7, is not in the CI, we can reject the null hypothesis at the 10% level.

Same conclusion as in part a.

4. A manufacturer claims in their TV advertisement that a new design for a portable phone has increased the range to 150 feet, allowing customers to use the phone throughout their homes and yards. A consumer protection agency believes that it's a false advertisement, suspecting that the range is less than 150 feet. They ask an independent testing laboratory to test the manufacturer's claim, and find that a random sample of 44 of these phones worked over an average distance of 142 feet, with a standard deviation of 12 feet.

a. Is there enough evidence to say that the manufacturer's claim is false?

$$H_0: \mu = 150 \qquad H_a: \mu < 150$$

Conditions: SRS, $n > 30$

Test statistics and p-value: (Using STAT → TESTS → 2: TTest (since sigma is unknown) with $\mu_0 = 150$, $\bar{x} = 142$, $S_x = 12$ and $n = 44$.)

$$t = -4.422$$

$$p\text{-value} = 3.27 \times 10^{-5} = 0.0000327$$

Since the p-value is very small, we have strong evidence to say that the manufacturer's claim is false. We can reject their claim that the mean range is 150 feet and conclude that it's less than 150 feet.

b. Can you use a confidence interval to test the claim?

No, not here since the test is a one-sided test. (We have "<" in the alternative hypothesis). We can only use a confidence interval to test a claim for two-sided tests.

Hypothesis Testing for a Population Mean or Population Proportion

1. A battery manufacturer advertises that the mean reserve capacity of a certain battery is 1500 hours. You suspect that the batteries' reserve time is less than the advertised value. To test this claim, you randomly select a sample of 50 batteries and find the mean reserve capacity to be 1424 hours. Assume that the population standard deviation is 320 hours. At $\alpha = 5\%$, do you have enough evidence to support the manufacturer's claim? At the 1% level? Can you test the claim using an appropriate CI? Explain. If your answer is yes, test the claim that way, too.

We did this in class.

2. The diameter of a spindle in a small motor is supposed to be 5 mm. If the spindle is either too small or too large, the motor will not perform properly. The manufacturer measures the diameter in a sample of 60 motors to determine whether the mean diameter has moved away from the target. They found the mean diameter of the sample to be 4.951 mm. Form past studies it is known that the population standard deviation is 0.2 mm. At $\alpha = 5\%$, do you have enough evidence to conclude that the mean diameter is not 5mm? At the 10% level?

$$H_0: \mu = 5 \text{ mm}$$

$$H_a: \mu \neq 5 \text{ mm}$$

Conditions: SRS, $n > 30$

Test statistics and p-value: (Using STAT → TESTS → 1: ZTest (since sigma is known) with $\mu_0 = 5$, $\bar{x} = 4.951$, $\sigma = 0.2$ and $n = 60$.)

$$z = -1.898$$

$$p\text{-value} = 0.0578$$

At the 5% significance level we do not have enough evidence to reject the null hypothesis, although the p-value is very close to 5%. We can't reject the claim that the mean diameter of the spindles is 5 mm.

At the 10% significance level we have enough evidence to reject the null hypothesis. That is, we can reject the claim that the mean diameter of the spindles is 5mm, and conclude that it's not 5 mm.

Can you test the claim using an appropriate CI? Explain. If your answer is yes, test the claim that way, too.

Yes, we can since we have a two-sided test (" \neq " in the alternative hypothesis.)

Using STAT \rightarrow TESTS \rightarrow 7: ZInterval is (4.9004, 5.0016). Since the claimed value, 5, is inside the CI (barely, though) we cannot reject it. Same conclusion we had at the 5% significance level above.

The 90% CI is (4.9085, 4.9935). Since the claimed value, 5, is NOT inside the CI, we can reject it. Same conclusion we had at the 10% significance level above.

3. The number of years it took a random sample of 33 former smokers to quit permanently is listed. We know that the population standard deviation is 1.5 years. A health agency claims that the mean time it takes smokers to quit smoking permanently is 6 years. You think it's more than that. At $\alpha=5\%$, is there enough evidence to reject the health agency's claim? At the 1% level? Can you test the claim using an appropriate CI? Explain. If your answer is yes, test the claim that way, too.

10.7	3.2	12.6	9.0	8.7	9.2	0.7	3.0	2.3	8.2	12.8
9.8	3.3	1.8	0.8	1.0	2.5	8.1	20.7	12.5	9.9	2.9
7.1	8.3	5.2	7.0	7.8	0.1	3.5	10.5	4.2	1.1	6.1

$H_0: \mu = 6$ years $H_a: \mu > 6$ years

Conditions: SRS, $n > 30$

Test statistics and p-value:

First you enter all these numbers into one of the lists in your calculator: STAT \rightarrow EDIT \rightarrow 1:Edit... Let's say you enter all of these numbers into L1.

Then use STAT \rightarrow TESTS \rightarrow 1: ZTest (since sigma is known) and highlight **Data** and enter $\mu_0 = 6$, $\sigma = 1.5$, List: L1, Freq: 1.

$z = 1.9264$ $p\text{-value} = 0.027$

At the 5% significance level we have enough evidence to reject the null hypothesis since the p-value is less than 5%. We can reject the claim that the mean time it takes smokers to quit is 6 years, and we can conclude that it's more than 6 years.

At the 1% significance level we do not have enough evidence to reject the null hypothesis. That is, we cannot reject the claim that the mean time it takes smokers to quit is 6 years.

We can't use a CI to test the claim because this is not a two-sided test.

4. An industrial company claims that the mean pH level of the water in a nearby river is 6.8. You randomly select 19 water samples and measure the pH level of each. The sample mean and standard deviation are 6.7 and 0.24 respectively. Is there enough evidence to reject the company's claim at $\alpha = 0.05$? Assume that the population is normally distributed. Is your conclusion the same at $\alpha = 0.10$ level?
Can you test the claim using an appropriate CI? Explain. If your answer is yes, test the claim that way, too.

$$H_0: \mu = 6.8 \qquad H_a: \mu \neq 6.8$$

Conditions: SRS, normal distribution

Test statistics and p-value: (Using STAT \rightarrow TESTS \rightarrow 2: TTest (since sigma is unknown) with $\mu_0 = 6.8$, $\bar{x} = 6.7$, $S_x = 0.24$ and $n = 19$.)

$$t = -1.82 \qquad p\text{-value} = 0.086$$

At the 5% significance level we do not have enough evidence to reject the null hypothesis since the p-value is greater than 5%. That is, we cannot reject the claim that the pH level of the water in the nearby river is 6.8.

At the 10% significance level we have enough evidence to reject the null hypotheses since the p-value is less than 10%. That is, we can conclude that the pH level of the water in the nearby river is not 6.8.

Yes, we can use a CI to test the claim because it's a two sided test (we have a " \neq " in the alternative hypothesis). Using STAT \rightarrow TESTS \rightarrow 8: TInterval the 95% CI is (6.5843, 6.8157). Since the claimed value, 6.8, is in the confidence interval, we can't reject the claim at the 5% level.

The 90% is (6.6045, 6.7955). Since the claimed value, 6.8, is not in the CI, we can reject the null hypothesis at the 10% level.

Same conclusion as above.

5. You receive a brochure from a large university. The brochure indicates that the mean class size for full-time faculty is 32 students. When you visit the classes at the university, you have a feeling that the class sizes are more than 32. You want to test the universities claim, so you randomly select 18 classes taught by full-time faculty and determine the class size of each. The results are listed below. At $\alpha = 0.01$ level, can you support the university's claim? How about at $\alpha = 0.05$ level? Assume that the population is normally distributed.
Can you test the claim using an appropriate CI? Explain. If your answer is yes, test the claim that way, too.

44 34 29 33 35 40 26 37 29 34 30 36 38 34 27 30 28 25

$$H_0: \mu = 32 \qquad H_a: \mu > 32$$

Conditions: SRS, normal distribution

Test statistics and p-value:

First you enter all these numbers into one of the lists in your calculator: STAT → EDIT → 1:Edit...
Let's say you enter all of these numbers into L2.

Then use STAT → TESTS → 1: TTest (since sigma is unknown) and highlight **Data**
and enter $\mu_0 = 32$, List: L2, Freq: 1.

$$t = 0.596 \quad p\text{-value} = 0.28$$

Both at the 1% and 5% significance level we do not have enough evidence to reject the null hypothesis since the p-value is greater than 5%. That is, we cannot reject the claim that the mean class size at that university is 32. This test supports the university's claim.

We can't use a CI to test the claim because this is not a two-sided test.

6. The manufacturer of a certain energy bar advertises that their energy bar contains 14 g of protein. You work for a nutritional health agency and are asked to test this claim because many consumers believe that the protein level is lower. You find that a random sample of 60 energy bars have mean protein level of 13.82 g and standard deviation 1.6 g. At the 10% level, do you have enough evidence to reject the manufacturer's claim?

$$H_0: \mu = 14 \text{ g} \quad H_a: \mu < 14 \text{ g}$$

Conditions: SRS, $n > 30$

Test statistics and p-value: (Using STAT → TESTS → 2: TTest (since sigma is unknown) with $\mu_0 = 14$, $\bar{x} = 13.82$, $S_x = 1.6$ and $n = 60$.)

$$t = -0.871 \quad p\text{-value} = 0.194$$

At the 10% significance level we do not have enough evidence to reject the null hypotheses since the p-value is greater than 10%. That is, we cannot reject the manufacturer's claim that the mean protein level in their energy bar is 14g.

Can you test the claim using an appropriate CI? Explain. If your answer is yes, test the claim that way, too.

We can't use a CI to test the claim because this is not a two-sided test.

7. Employees in a large accounting firm claim that the mean salary of the firm's accountants is less than that of its competitor's which is \$45,000. A random sample of 35 of the firm's accountants

has a mean salary of \$43,500. Assume that the population standard deviation is \$5200. At $\alpha=5\%$, test the employees claim. Do you have the same conclusion at the 1% level?

$$H_0: \mu = \$45,000 \qquad H_a: \mu < \$45,000$$

Conditions: SRS, $n > 30$

Test statistics and p-value: (Using STAT \rightarrow TESTS \rightarrow 1: ZTest (since sigma is known) with $\mu_0 = 45000$, $\sigma=5200$, $\bar{x} = 43500$, and $n = 35$.)

$$z = -1.7 \qquad \text{p-value} = 0.044$$

At the 5% significance level we have enough evidence to reject the null hypothesis since the p-value is less than 5%. That is, we can reject the large accounting firm's claim that the mean salary of the firm's accountants is \$45,000. We can conclude that the mean salary is less than \$45,000.

At the 1% level we do not have enough evidence to reject the null hypothesis since the p-value is greater than 1%. Thus, at the 1% level, we can't reject the claim that the mean salary is \$45,000.

8. In Illinois, a random sample of 85 eighth grade students has a mean score of 265 on a national mathematics assessment test. This test result prompts a state school administrator to declare that the mean score for the state's eighth graders on the examination is more than the national average, 260. Assume that $\sigma=35$. At $\alpha=5\%$, is there enough evidence to support the administrator's claim? Do you have the same conclusion at the 1% level? At the 10% level?

$$H_0: \mu = 260 \qquad H_a: \mu > 260$$

Conditions: SRS, $n > 30$

Test statistics and p-value: (Using STAT \rightarrow TESTS \rightarrow 1: ZTest (since sigma is known) with $\mu_0 = 260$, $\sigma=35$, $\bar{x} = 265$, and $n = 85$.)

$$z = 1.371 \qquad \text{p-value} = 0.094$$

Both at the 5% and 1% significance level we do not have enough evidence to reject the null hypothesis since the p-value is greater than 5%. And 1% That is, we cannot reject the claim that the mean score on the national assessment test is 260.

At the 10% level we do have some evidence (not strong) to reject the null hypothesis since the p-value is less than 10%. Thus, at the 10% level, we can reject the claim that the mean score on the national mathematics assessment test is 260, and can conclude that it is more than 260.

9. A coffee shop claims that its fresh-brewed drinks have a mean caffeine content of 80 milligrams per 5 ounces. You work for a city health agency and are asked to test this claim. You find that in a random sample of 42 five-ounce servings has a mean caffeine content of 83 milligrams per 5

ounces. Assume that $\sigma=11$ milligrams. At $\alpha=5\%$, is there enough evidence to reject the shop's claim? Do you have the same conclusion at the 1% level? At the 10% level?
 Show that you can reach the same conclusion using confidence intervals.

$$H_0: \mu = 80 \qquad H_a: \mu \neq 80$$

Conditions: SRS, $n > 30$

Test statistics and p-value: (Using STAT \rightarrow TESTS \rightarrow 1: ZTest (since sigma is known) with $\mu_0 = 80$, $\sigma=11$ $\bar{x} = 83$, and $n = 42$.)

$$z = 1.77 \qquad p\text{-value} = 0.077$$

Both at the 5% and 1% significance level we do not have enough evidence to reject the null hypothesis since the p-value is greater than 5% and 1%. That is, we cannot reject the coffee shop's claim that the mean caffeine content in their fresh-brewed coffee is 80 milligrams per 5 oz.
 At the 10% level we have enough evidence to reject the null hypothesis since the p-value is less than 10%. Thus, at the 10% level, we can reject the coffee shop's claim, and conclude that the mean caffeine content in their fresh-brewed coffee is not 80 milligrams per 5 oz, but we don't have strong evidence against their claim, just some evidence.

We can test the claim using CIs since it's a two-sided test we have a " \neq " in the alternative hypothesis). Using STAT \rightarrow TESTS \rightarrow 7: ZInterval the 95% CI is (79.673, 86.327). Since the claimed value, 80, is in the confidence interval, we can't reject the claim at the 5% level.

The 99% confidence interval is (78.628, 87.372). Since the claimed value, 80, is inside the CI, we can't reject the coffee shop's claim at the 1% level.

The 90% is (80.208, 85.792). Since the claimed value, 80, is not in the CI, we can reject the null hypothesis at the 10% level.

Same conclusions as above.

10. A researcher claims that 15% of adults in the U.S. are allergic to a trees, weeds, flowers, and grasses. You think that more than 15% are allergic to these. In a random sample of 86 adults, 17 say they have such an allergy. Test the researcher's claim at $\alpha = 5\%$ level. Can you reach the same conclusion using the appropriate confidence interval?

$$H_0: p = 0.15 \qquad H_a: p > 0.15$$

$$\hat{p} = \frac{17}{86} = 0.198$$

$$\text{Conditions: SRS, } n\hat{p} = 86(0.198) = 17 > 10 \qquad n(1 - \hat{p}) = 86(1 - 0.198) = 69 > 10$$

Test statistics and p-value: (Using STAT → TESTS →5: 1-PropZTest with $p_0 = 0.15$, $x = 17$, and $n = 86$.)

$$z = 1.238 \qquad p\text{-value} = 0.108$$

At the 5% level we don't have enough evidence to reject the null hypothesis since the p-value is greater than 5%. That is we cannot reject the claim that 15 % of adults in the U.S. are allergic to a trees, weeds, flowers, and grasses.

We can't use a CI to test the claim because this is not a two-sided test.

11. The Pew Research Center claims that 55% of U.S. adults regularly watch a network news broadcast. You think that they overestimate this proportion, so you ask a random sample of 425 adults in the United States whether they regularly watch a network news broadcast. Of the 425 adults, 215 responded yes. At the 5% significance level, is there enough evidence to support the claim? How about at the 1% significance level?

$$H_0: p = 0.55 \qquad H_a: p < 0.55$$

$$\hat{p} = \frac{215}{425} = 0.506$$

$$\text{Conditions: SRS, } n\hat{p} = 425(0.506) = 215 > 10 \qquad n(1 - \hat{p}) = 425(1 - 0.506) = 210 > 10$$

Test statistics and p-value: (Using STAT → TESTS →5: 1-PropZTest with $p_0 = 0.55$, $x = 215$, and $n = 425$.)

$$z = -1.83 \qquad p\text{-value} = 0.034$$

At the 5% level we have enough evidence to reject the null hypothesis since the p-value is less than 5%. That is we can reject the claim that 55% of U.S. adults regularly watch a network news broadcast and conclude that less than 55% watch those.

At the 1% level we don't have enough evidence to reject the null hypothesis since the p-value is greater than 1%. That is we cannot reject the claim that 55 % of adults in the U.S. watch a network news broadcast regularly.

12. An environmentalist claims that 58% of British consumers want supermarkets to stop selling genetically modified foods. You want to test this claim. You find that in a random sample of 100 British consumers, 61% say that they want supermarkets stop selling genetically modified foods. At the 10% significance level, can you support the environmentalist's claim.

$$H_0: p = 0.58 \qquad H_a: p \neq 0.58$$

$$\hat{p} = 61\% = 0.61$$

$$\text{Conditions: SRS, } n\hat{p} = 100(0.61) = 61 > 10 \qquad n(1 - \hat{p}) = 100(1 - 0.61) = 39 > 10$$

Test statistics and p-value: (Using STAT → TESTS → 5: 1-PropZTest with $p_0 = 0.58$, $x = 61$, and $n = 100$. ($x = 100(0.61) = 61$)

$$z = 0.608$$

$$p\text{-value} = 0.54$$

At the 10% level we don't have enough evidence to reject the null hypothesis since the p-value is greater than 10%. That is we cannot reject the environmentalist's claim that 58 % of British consumers want supermarkets to stop selling genetically modified foods.