M 225   **Test 2 B**   Name_____

SHOW YOUR WORK FOR FULL CREDIT!

Problem	Max. Points	Your Points
1-10	10	
11	10	
12	6	
13	6	
14	5	
15	12	
16	4	
17	6	
18	6	
19	3	
20	3	
21	3	
22	6	
23	13	
24	6	
Total	100	

Multiple choice questions (1 point each)

1. A correct interpretation of the statement "The probability that a child delivered in a certain hospital is a girl is 0.50" would be which one of the following?
   a. In the next two births at that hospital, there will be exactly one boy and one girl.
   b. Over a long period of time, there will be equal proportions of boys and girls born at that hospital.
   c. To make sure that a couple has two girls and two boys at that hospital, they only need to have four children.
   d. A computer simulation of 100 births for that hospital would produce exactly 50 girls and 50 boys.

2. From a computer simulation of rolling a fair die ten times, the following data were collected on the showing face: 5   1   5   1   6   2   5   3   5   1
   What is a correct conclusion to make about the next ten rolls of the same die?

   a. The probability of rolling a 4 is 0, and therefore we will not roll it in the next ten rolls.
   b. The probability of rolling a 5 is greater than the probability of rolling anything else.
   c. Each face has exactly the same probability of being rolled.
   d. We will see exactly three faces showing a 1 since it is what we saw in the first experiment.

3. If a couple wants to have three children. Let X represent the number of girls. What is the probability that the couple will have one girl and two boys?

 | X | 0 | 1 | 2 | 3 |
   |------|-----|-----|-----|-----|
   | P(X) | 1/8 | 3/8 | 3/8 | 1/8 |

   d. 3/8
   e. 1/8
   f. 1/8 + 1/8 = 2/8
   g. 1/8+3/8 = 4/8

4. Three students work independently on a homework problem. The probability that the first student solves the problem is 0.95. The probability that the second student solves the problem is 0.85. The probability that the third student solves the problem is 0.80.
   What is the probability that all are able to solve the problem?
   a. (0.95) (0.85) (0.80)
   b. 0.95 + 0.85 + 0.80
   c. 1 - (0.95) (0.85) (0.80)
   d. 0.80

5. A movie theater sells 3 sizes of popcorn (small, medium, and large) with 3 choices of toppings (no butter, butter, extra butter). How many possible ways can a bag of popcorn be purchased?
   a. 12
   b. 6
   c. 27
   d. 9

6. A manufacturing company takes a sample of $n = 100$ bolts from their production line. X is the number of bolts that are found defective in the sample. It is known that the probability of a bolt being defective is 0.003.
Does X have a binomial distribution?
   a. Yes.
   b. No, because there is not a fixed number of observations.
   c. No, because the observations are not all independent.
   d. No, because there are more than two possible outcomes for each observation.
   e. No, because the probability of success for each observation is not the same.

7. A survey-taker asks the age of each person in a random sample of 20 people. X is the age for the individuals.
Does X have a binomial distribution?
   a. Yes.
   b. No, because there are more than two possible outcomes for each observation.
   c. No, because there is not a fixed number of observations.
   d. No, because the observations are not all independent.

8. A fair die is rolled and the number of dots on the top face is noted. X is the number of times we have to roll in order to have the face of the die show a 2.
Does X have a binomial distribution?
   a. Yes.
   b. No, because there are more than two possible outcomes for each observation.
   c. No, because there is not a fixed number of observations.
   d. No, because the observations are not all independent.
   e. No, because the probability of success for each observation is not the same.

9. Suppose that for a randomly selected high school student who has taken a college entrance exam, the probability of scoring above a 650 is 0.30. A random sample of $n = 9$ students was selected. What are the mean μ and standard deviation σ of the number of students in the sample who have scores above 650?
   a. $\mu = (9)(0.3) = 2.7, \sigma = 0.30$
   b. $\mu = 3, \sigma = (9)(0.3)$
   c. $\mu = (9)(0.3) = 2.7, \sigma = (9)(0.7)(0.3)$
   d. $\mu = (9)(0.3) = 2.7, \sigma = \sqrt{(9)(0.7)(0.3)}$

10. Which one of these pairs of events is a pair of disjoint (mutually exclusive) events?
   a. has never been married; has divorced
   b. has been in a foreign country; has never ridden on a train
   c. owns a piano; owns a violin
   d. has brown hair; has blue eyes

11. TRUE or FALSE (1 point each)

(T) **F** Whenever the population has a normal distribution, the sampling distribution of sample means is normally distributed.

T **(F)** These two normal curves have the same standard deviations, but different means.
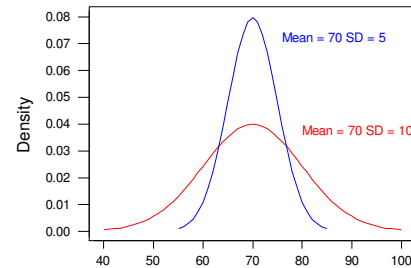
(T) **F** The probability that a person has a certain disease is 0.04. Then the probability that two randomly selected persons do NOT have the disease is $(1-0.04)^2$.

T **(F)** The Standard Normal Distribution has mean 1 and standard deviation 0.

(T) **F** For a normal distribution the z-score is 0 if the corresponding x-value is equal to the mean.

(T) **F** The sampling distribution of refers to the distribution of the different possible values of the sample mean or sample proportion

(T) **F** The area under any normal curve is 1.

(T) **F** When the z-score of a data value is −1.8, that means that the data value is 1.8 standard deviations below the mean.

T **(F)** An appropriate notation for the fraction of all American adults who received at least one speeding ticket last year is $\hat{p}$.

(T) **F** An appropriate notation for the mean distance traveled in a year by a sample of truck drivers is $\bar{x}$.

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12. (6 points) You roll two fair six-sided dice. Find the probability for each event:

a. Event A: the sum of the two numbers is 9

Sum of 9:   4,5; 5,4; 3,6; 6,3. Four possibilities out of 36. So the probability is 4/36

b. Event B: the sum is 4 or getting doubles

Sum of 4: 1,3; 3,1; 2,2. Doubles: 1,1; 2,2: 3,3; 4,4; 5,5; 6,6;
The event 2,2 was counted twice
So the probability is 3/36 + 6/36 – 1/36 =8/36

13. (6 points) Identify the sample space.

a.   A coin is tossed, and then a fair six-sided die rolled.

   S={T1, T2, T3, T4, T5, T6, H1, H2, H3, H4, H5, H6}

b.   You have three coins, a quarter (Q), a dime (D), and a nickel (N) in your pocket. You randomly pick one coin, and then without replacing it you select another coin.

   S={QD, QN, DQ, DN, NQ, ND}

14. (5 points) Decide whether the random variable *X* is discrete or continuous:

a.   *X* represents the length of time it takes to get to work Continuous

b.   *X* represents the number of highway fatalities in one week in LA county. Discrete

c.   *X* represents the weight of a chemical compound. Continuous

d.   *X* represents the volume of blood drawn for a blood test. Continuous

e.   *X* represents the number of books sold per semester at a university book store. Discrete

15. (12 points) During one season, the Los Angeles Dodgers had won 41 and lost 37 games. The breakdown by whether the game was played during the day or at night is shown in the following table. If you select a game randomly

		Won the Game?		
		Yes	No	Total
Time of	Day	10	11	21
Game	Night	31	26	57
	Total	41	37	78

a.   what is the probability that the team had not won that game?

   37 / 78

b.   what is the probability that the team had not won that game and played at night?

   11 / 78

c.   what is the probability that the team had won that game given that they played at night?

   31 / 57

d. what is the probability that the team played during the day given that they lost?

11 / 37

e. what is the probability that the team had not won that game given that they played during the day?

10 / 21

f. Are the events "Played during the day" and "Had Not Won the game" independent? Explain using the probabilities in parts (a) and (e).

Since the probabilities in parts (a) and (e) are not equal, these two events are not independent.

16. (4 points) The probability that a statistics student reads the textbook is 0.66. The probability a student does the homework is 0.74. The probability that a student reads the book and does the homework is 0.51.
If a student is randomly selected, what is the probability that the student reads the book or does the homework?

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0.66 + 0.74 - 0.51 = 0.89$

17. (6 points) A printing company's bookbinding machine has a probability of 0.018 of producing a defective book. This machine is used to bind three books.
a. Find the probability that all three books are defective.

$(0.018)(0.018)(0.018) = 5.832 \times 10^{-6}$

b. Find the probability that none of the three books are defective.

$(1 - 0.018)(1 - 0.018)(1 - 0.018) = 0.947$

18. (6 points) A sociologist surveyed the households in a small town. The random variable represents the number of dependent children per household.

X	0	1	2	3	4
P(X)	0.06	0.20	0.37		0.13

a. Find the missing probability value.

Since the probabilities must add up to 1, the missing value must be 0.24.

b. What is the probability that a randomly selected household has 3 or more dependent children?

3 or 4 children: 0.24 + 0.13 = 0.37

c. Find the mean number of dependent children per household.

Mean: $0(0.07) + 1(0.20) + 2(0.36) + 3(0.24) + 4(0.13) = 2.16$

19. (3 points) From a pool of 10 candidates, the offices of president, vice-president, secretary, and treasurer will be filled. In how many ways can the offices be filled?

Order DOES matter here, so it's $_{10}P_4 = 5040$

20. (3 points) Find the number of ways a four-member committee can be chosen from 10 candidates.

Order does NOT matter here, so it's $_{10}C_4 = 210$

21. (3 points) In how many distinguishable ways can the letters in the word MATHEMATICS?

11 letters in the word, M repeats 2 times, A repeats 2 times, and T repeats 2 times:
$(11!)/(2!)(2!)(2!) = 4989600$

22. (6 points) 53% of men consider themselves basketball fans. You randomly select 3 men and ask each if he considers himself a basketball fan.
a. Verify that the experiment is a binomial experiment.

n is fixed (n = 3), the trials are independent, the probability that a man is a basketball fan is constant, and there are only two possible outcomes (either a fan or not). So it's a binomial experiment.

b. What is the probability that out of the three men exactly two men consider themselves a basketball fan?

$$P(X = x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$P(X = 2) = \frac{3!}{2!(3-2)!} (0.53)^2 (1-0.53)^{3-2} = 0.3961$$

c. Find the mean and the standard deviation of the distribution.

Mean = np= $3(0.53) = 1.59$

Standard deviation = $\sqrt{np(1-p)} = \sqrt{3(0.53)(1-0.53)} = 0.864$

23. (13 points) The mean total cholesterol levels for men ages 35-44 are normally distributed with mean cholesterol level 202 milligrams per deciliter with a standard deviation of 39.1 milligrams per deciliter. (Don't forget to draw the distribution and shade the area.)

a. What is the probability that a randomly selected man in this age group has total cholesterol level below 120 milligrams per deciliter?

$$z = \frac{x - \mu}{\sigma} = \frac{120 - 202}{39.1} = -2.10 \qquad \text{probability: 0.018}$$

The probability that a randomly selected man in this age group has total cholesterol level below 120 milligrams per deciliter is about 1.8%.

b. What is the cholesterol level of men ages 35-44 who are in the top 5% ?

For the top 5%, the z-score is 1.645 (INSIDE the table look for the number closest to 0.95)

x = z·σ + μ = 1.645(39.1) + 202 = 266.32

The cholesterol level of men ages 35-44 in the top 5% is 266.32 milligrams per deciliter or more.

c. If we would create the sampling distribution the sample means for all possible samples of size 100 for. Using the Central Limit Theorem determine the shape, center, and standard deviation of the distribution

Shape: normal since the sample size 100 > 30

Mean: same as the population mean: 202

Standard deviation: $\frac{\sigma}{\sqrt{n}} = \frac{39.1}{\sqrt{100}} = 3.91$

d. What is the probability that the mean cholesterol level of a random sample of 100 men in this age group is more than 215 milligrams per deciliter?

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{215 - 205}{3.91} = 2.56$$

probability: 0.0053

The probability that a randomly sample of 100 men in this age group has total mean cholesterol level 215 milligrams per deciliter or more is about 0.53%.

24. (6 points) During a certain week the mean price of gasoline (87) in CA was $3.09 per gallon with standard deviation of $0.08.

a. What is the probability that a randomly selected gas station in CA has gasoline (87) price less than $3.00?
We can't answer to this one. We don't know anything about the population distribution.

b. What is the probability that the mean gasoline (87) price of 18 randomly selected gas stations is more than $3.25?

We can't answer to this one. We don't know anything about the population distribution, and 18 is not big enough to use the CLT.

c. What is the probability that the mean gasoline (87) price of 38 randomly selected gas stations is between $3.10 and $3.12?

Now we can calculate the probability because n >30. We can find the two z-scores with mean 3.09 and standard deviation $\dfrac{\sigma}{\sqrt{n}} = \dfrac{0.08}{\sqrt{38}} = 0.013$

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{3.10 - 3.09}{0.013} = 0.77$$

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{3.12 - 3.09}{0.013} = 2.31$$

The probability is: 0.9896 – 0.7794 = 0.2102

Thus, the probability that the mean gasoline price of 38 randomly selected gas stations is between $3.10 and $3.12 is about 21%.