

SHOW YOUR WORK FOR FULL CREDIT!

Problem	Max. Points	Your Points
1-10	10	
11	10	
12	10	
13	6	
14	11	
15	3	
16	4	
17	6	
Total	60	

1. Suppose you conduct a significance test for a population proportion using $\alpha=10\%$ and your p-value is .184. Which of the following should be your conclusion?

- a. Accept H_0
- b. Accept H_A
- c. Fail to reject H_A
- ☒ d. Fail to reject H_0

2. When are p-values negative?

- a. when the test statistic is negative.
- b. when the sample statistic is smaller than the hypothesized value of the parameter
- c. when the confidence interval includes only negative values
- d. when we fail to reject the null hypothesis
- ☒ e. never

3. You have measured the systolic blood pressure of a random sample of 25 employees of a company. A 95% confidence interval for the mean systolic blood pressure for the employees is computed to be (122,138). Which of the following statements gives a valid interpretation of this interval?

- a. About 95% of the sample of employees have a systolic blood pressure between 122 and 138.
- b. About 95% of the employees in the company have a systolic blood pressure between 122 and 138.
- ☒ c. If the sampling procedure were repeated many times, then approximately 95% of the resulting confidence intervals would contain the mean systolic blood pressure for employees in the company.
- d. If the sampling procedure were repeated many times, then approximately 95% of the sample means would be between 122 and 138.
- e. The probability that the sample mean falls between 122 and 138 is equal to 0.95.

4. The average growth of a certain variety of pine tree is 10.1 inches in three years. A biologist claims that a new variety will have a greater three-year growth. A random sample of 45 of the new variety has an average three-year growth of 10.8 inches and a standard deviation of 2.1 inches.

The appropriate null and alternate hypotheses to test the biologist's claim are:

- a. $H_0: \mu = 10.8$ against $H_a: \mu > 10.8$
- b. $H_0: \mu = 10.8$ against $H_a: \mu \neq 10.8$
- ☒ c. $H_0: \mu = 10.1$ against $H_a: \mu > 10.1$
- d. $H_0: \mu = 10.1$ against $H_a: \mu < 10.1$
- e. $H_0: \mu = 10.1$ against $H_a: \mu \neq 10.1$

5. What is statistical inference on μ ?

- ☒ a. Drawing conclusions about a population mean based on information contained in a sample.
- b. Drawing conclusions about a sample mean based on information contained in a population.
- c. Drawing conclusions about a sample mean based on the measurements in that sample.
- d. Drawing conclusions about the population proportion based on information contained in the sample.

6. You take a random sample from some population and form a 95% confidence interval for the population mean, μ . Which quantity is guaranteed to be in the interval you form?

- a. p
- b. μ
- ☒ c. $\bar{x}$
- d. 0.95
- e. $\hat{p}$

7. The 90% confidence interval for a population mean is (1.2, 5.2)

- a. Then the population mean is 3.2, and the margin of error is 2.
- ☒ b. Then the sample mean is 3.2, and the margin of error is 2.
- c. Then the sample mean is 2, and the margin of error is 3.2.
- d. Then the sample mean is 1.2, and the margin of error is 5.2.

8. Which of the following is true about p-values?

- ☒ a. The p -value for a specific statistical test is the probability (assuming H_0 is true) that the test statistic will take a value at least as extreme as that actually observed.
- b. The p -value for a specific statistical test is the probability (assuming H_0 is true) that alternative hypothesis is true.
- b. The p -value for a specific statistical test is the probability (assuming H_a is true) that the test statistic will take a value at least as extreme as that actually observed.
- c. All of the above statements are true.
- d. None of the above statements are true.

9. An appropriate 95% confidence interval for μ has been calculated as (-0.73, 1.92) based on $n=15$ observations from a population with a normal distribution. Suppose we wish to test $H_0 : \mu = 0$ versus $H_a: \mu \neq 0$. Based on this confidence interval,

- a. we should reject H_0 at the $\alpha = 0.05$ level of significance.
- ☒ b. we should not reject H_0 at the $\alpha = 0.05$ level of significance.
- c. we should reject H_0 at the $\alpha = 0.10$ level of significance.
- d. we should not reject H_0 at the $\alpha = 0.10$ level of significance.

10. Which one of these statements is FALSE?

- a. Increasing the sample size will decrease the margin of error.
- ☒ b. Increasing the confidence level will decrease the margin of error.
- c. It is usually unrealistic to assume that we know the population standard deviation.
- d. Confidence intervals are not valid if the sample is not chosen randomly.

11. In a random sample of 42 patients at a hospital's emergency department, the mean waiting time (in minutes) before seeing a medical professional was calculated along with the sample standard deviation.

- a. In this study, what is the parameter we want to estimate (in context)?

The mean waiting time (in minutes) for ALL patients before seeing a medical professional at this emergency department.

- b. Based on these sample results, a 95% confidence interval for the parameter of interest was calculated: (19.57, 26.43). Interpret your results in context.

We are 95% confident that the mean waiting time for ALL patients before seeing a medical professional at this emergency department is between 19.57 minutes and 26.43 minutes.

- c. For the following statements about the confidence interval given above, decide whether they are TRUE or FALSE?

- ☐ T ☒ F If we had sampled 50 patients instead of 42, the margin of error would have been less.
- T ☒ F A 90% confidence interval with the same data would contain 27 minutes.
- ☒ T ☐ F The mean waiting time for this sample of 42 patients was 23 minutes.
- ☒ T ☐ F In repeated sampling, about 95% of all intervals computed from samples of the same size will contain the true population parameter.
- T ☒ F 95% of the 42 patients had to wait between 19.57 and 26.43 minutes.

12. A recent study claims that 23% of people in the U.S. are in favor of outlawing cigarettes. A health advocacy group claims that the proportion of people in the U.S. who are in favor of outlawing cigarettes is higher than the study's claim, so they decide to test this claim and ask a random sample of 200 people in the U.S. whether they are in favor of outlawing cigarettes. Of the 200 people, 54 are in favor.

- a. Specify the null and alternative hypotheses for this test, using the correct symbols and numbers.

Null: $p = 0.23$

Alternative: $p > 0.23$

- b. Check the conditions for a hypothesis test.

SRS, checked.

$$np_0 = 200(0.23) = 54 > 10$$

$$n(1 - p_0) = 200(1 - 0.23) = 144 > 10$$

- c. Determine the value of the test statistic, and the p-value.

$$z = 1.34$$

$$p\text{-value} = 0.089$$

- d. Which one of the following statements is true at the 0.05 level of significance?
- (i) the results are significant, and so we can reject the null hypothesis.
 - (ii) the results are not significant, and so we can reject the null hypothesis.
 - (iii) the results are significant, and so we cannot reject the null hypothesis.
 - (iv) the results are not significant, and so we cannot reject the null hypothesis.
- e. State your conclusion in context. Make sure include a statement about the strength of the evidence against the 23% figure claimed in the study.

Since the p-value is between 5% and 10%, we have little evidence against the null hypothesis. At the 5% significance level, we can't reject the null hypothesis. That is, we can't reject the claim that 23% of people in the U.S. are in favor of outlawing cigarettes.

At the 10% significance level we can reject the null hypothesis. In this level, we can reject the claim that 23% of people in the U.S. are in favor of outlawing cigarettes.

13. A Bloomberg Poll conducted a telephone survey between Sept. 6-10, 2007 to estimate the percent of voters in South Carolina who would vote in the Democratic primary. Out of the 370 registered South Carolina voters contacted by the poll, 136 planned to vote in the Democratic primary. For the 95% confidence interval, the poll reported a margin of error of ± 5 percentage points.

- a. Carefully show how this margin of error was computed.

$$\hat{p} = \frac{x}{n} = \frac{136}{370} = 0.37 \qquad E = z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.96 \sqrt{\frac{0.37(1-0.37)}{370}} = 0.049 \approx 0.05 = 5\%$$

- b. For the next poll they want to reduce the margin of error to ± 3 percentage points. How many voters should they contact?

$$n = \left(\frac{z^*}{E} \right)^2 \hat{p}(1-\hat{p}) = \left(\frac{1.96}{0.03} \right)^2 0.37(1-0.37) = 994.9744$$

Therefore, they should contact at least 995 voters.

14. A certain cold medication must contain the amount of the active ingredient as **4 mg** per dose to be effective. If the medication contains more than 4mg per dose, it's dangerous to health. If it is less than 4mg per dose, the medication is not effective. A random sample of 55 doses was taken to a lab and analyzed to determine the actual amounts of the active ingredient in each of these 55 doses. They found **3.97 mg** active ingredients per dose with standard deviation 0.071 mg.

- a. Is the **4mg** a parameter or a statistics? Explain. Is the **3.97mg** parameter or a statistic? Explain.

4 mg is a parameter. That's what supposed to be the amount of the active ingredient for ALL medication. On the other hand, 3.97mg is the mean active ingredient of the sample, so it's a statistic. This is the sample mean.

- b. The medication won't be approved if the active ingredient is not 4mg. State the null and alternative hypotheses in symbols.

$$H_0: \mu = 4\text{mg}$$

$$H_a: \mu \neq 4\text{mg}$$

- c. Use the Minitab output below to test the claims. Use both outputs (T-Test of the Mean, and T Confidence Intervals) in your decision. Write your conclusions in context.

T-Test of the Mean

Variable	N	Mean	StDev	T	P
INGR	55	3.97	0.071	- 3.13	0.0028

T Confidence Intervals

Variable	N	Mean	StDev	95.0 % CI
INGR	55	3.97	0.071	(3.951, 3.989)

Since the p-value is less than 5%, we can reject the null hypothesis. Also, since the CLAIMED value, 4mg is NOT in the 95% CI, it's not a plausible value, so we can reject the null hypothesis. Same conclusion reached by two ways.

That is, we can reject the claim that the mean amount of active ingredient per dose in that medication is 4mg. We have enough evidence to conclude that the mean amount of the active ingredient per dose in that medication is NOT 4mg. Therefore, the medication should not be approved.

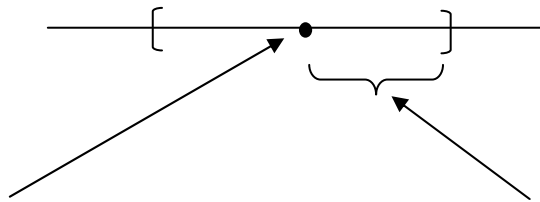
- d. Is the confidence interval valid if the distribution from which the 55 measurements were taken is not normally distributed? Explain briefly.

Yes, it's valid since the sample size is big, greater than 30. By the Central Limit Theorem we can be sure that the sampling distribution will be approximately normal.

15. A class of 54 students were asked to rate their statistics professor on the scale of 1-10 (1 meaning horrible, and 10 meaning excellent). The professor wants to know her mean score given by the class. Does it make sense here to compute a 95% confidence interval for the mean score? Explain.

No. Since the population is small, the professor can calculate the mean score. It doesn't need to be estimated using a sample from the class.

16. A confidence interval for a population proportion can be diagrammed on a number line as shown below. Identify (give the correct names for) the two items shown:



- (a) Sample proportion (b) margin of error

17. Explain what we mean by Type I and Type II errors in hypothesis testing.

Type I error: when the null hypothesis is true but we reject it.

Type II error: when the null hypothesis is not true but we don't reject it.