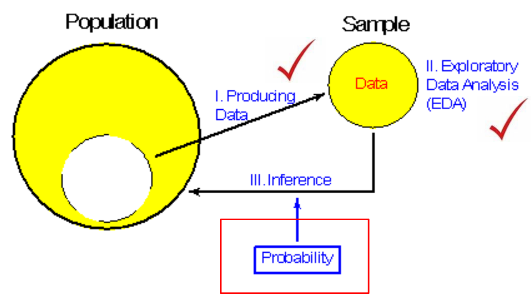
Chapter 4
Probability

Section 4-2: Fundamentals
Section 4-3: Addition Rule
Sections 4-4, 4-5: Multiplication Rule
Section 4-7: Counting (next time)

The Big Picture of Statistics



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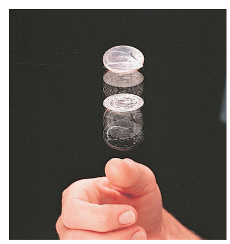
What is probability?

- ***Probability*** **is a mathematical description of randomness and uncertainty.**
- ***Random experiment*** is an experiment that produces an outcome that cannot be predicted in advance (hence the uncertainty).

Example: coin toss

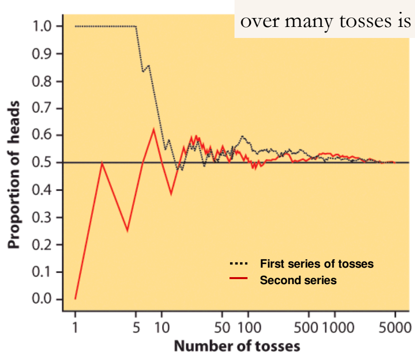
The result of any single coin toss is random.



Possible outcomes:
- Heads (H)
- Tails (T)

Coin toss

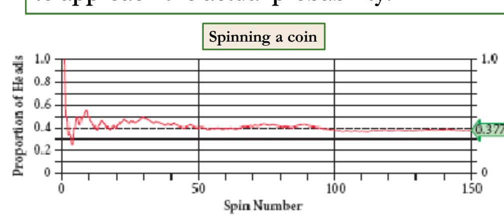
- The result of any single coin toss is random. But the result over many tosses is predictable.



- The probability of heads is 0.5 = the proportion of times you get heads in many repeated trials.

The Law of Large Numbers

As a procedure repeated again and again, the relative frequency probability of an event tends to approach the actual probability.



http://bcs.whfreeman.com/ips4e/cat_010/applets/expectedvalue.html

Sample Space

This list of possible outcomes an a random experiment is called ***the sample space*** of the random experiment, and is denoted by the letter ***S***.

Examples

- **Toss a coin once:** S = {H, T}.
- **Toss a coin twice:** S = {HH, HT, TH, TT}
- **Roll a dice:** S = {1, 2, 3, 4, 5, 6}
- **Chose a person at random and check his/her blood type:** S = {A,B,AB,O}.

Sample space

- **Important: It's the question that determines the sample space.**

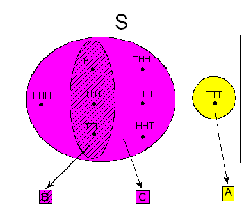
- **A basketball player shoots three free throws. What are the possible sequences of hits (H) and misses (M)?**
  - S = {HHH, HHM, HMH, HMM, MHH, MHM, MMH, MMM }
  - Note: 8 elements, 2^3
- **A basketball player shoots three free throws. What is the number of baskets made?**
  - S = {0, 1, 2, 3}

An Event

- **An *event* is an outcome or collection of outcomes of a random experiment.**
- Events are denoted by capital letters (other than S, which is reserved for the sample space).

- **Example:** tossing a coin 3 times. The sample space in this case is:
  S = {HHH, THH, HTH, HHT, HTT, THT, TTH, TTT}
- We can define the following events:
  Event A: "Getting no H"
  Event B: "Getting exactly one H"
  Event C: "Getting at least one H"

Example



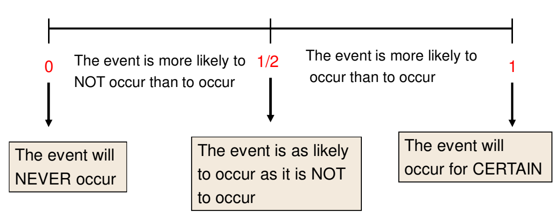
- *Event A:* "Getting no H" --> TTT
- *Event B:* "Getting exactly one H" --> HTT, THT, TTH
- *Event C:* "Getting at least one H" --> HTT, THT, TTH, THH, HTH, HHT, HHH

Probability

- Once we define an event, we can talk about the ***probability of the event*** happening and we use the notation:
- *P(A)* - the probability that event A occurs,
- *P(B)* - the probability that event B occurs, etc.

- ***The probability of an event tells us how likely is it for the event to occur.***

Probability of an Event



Equally Likely Outcomes

- If you have a list of all possible outcomes and all outcomes are **equally likely**, then the probability of a specific outcome is

$$\frac{1}{\text{the number of equally likely outcomes}}$$

Example: roll a die

- Possible outcomes: 

S={1,2,3,4,5,6}

Each of these are equally likely.

- Event A: rolling a 2

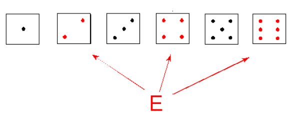
  The probability of rolling a 2 is P(A)=1/6

- Event B: rolling a 5

  The probability of rolling a 5 is P(A)=1/6

Example: roll a die

- Event E: getting an even number.



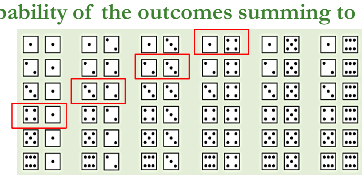
- Since 3 out of the 6 equally likely outcomes make up the event E (the outcomes {2, 4, 6}), the probability of event E is simply P(E)= 3/6 = 1/2.

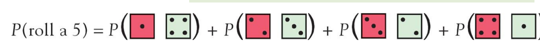
Example: roll two dice

What is the probability of the outcomes summing to five?

This is S:

{(1,1), (1,2), (1,3), ……etc.}





There are 36 possible outcomes in *S*, all equally likely (given fair dice). Thus, the probability of any one of them is 1/36.

***P*(sum is 5) = *P*(1,4) + *P*(2,3) + *P*(3,2) + *P*(4,1) = 4 * 1/36 = 1/9 = 0.111**

A couple wants three children. What are the arrangements of boys (B) and girls (G)?

Genetics tells us that the probability that a baby is a boy or a girl is the same, 0.5.

- Sample space: {BBB, BBG, BGB, GBB, GGB, GBG, BGG, GGG}
- All eight outcomes in the sample space are equally likely.
- The probability of each is thus 1/8.

A couple wants three children. What are the numbers of girls (*X*) they could have?

The same genetic laws apply. We can use the probabilities above to calculate the probability for each possible number of girls.

Sample space {0, 1, 2, 3}

- $P(X = 0) = P(BBB) = 1/8$
- $P(X = 1) = P(BBG \text{ or } BGB \text{ or } GBB) = P(BBG) + P(BGB) + P(GBB) = 3/8$

Value of *X*	0	1	2	3
Probability	1/8	3/8	3/8	1/8

Probability Rules

1. The probability P(A) for any event A is $0 \le P(A) \le 1$.
2. If S is the sample space in a probability model, then P(S)=1.
3. For any event A, P(A does not occur) = 1- P(A).

Examples

Rule 1: For any event A, $0 \le P(A) \le 1$

- Determine which of the following numbers could represent the probability of an event?
  - 0
  - 1.5
  - -1
  - 50%
  - 2/3

Examples

Rule 2: *P*(sample space) = 1

Blood type	O	A	B	AB
Probability	0.44	?	0.10	0.04

Blood type	O	A	B	AB
Probability	0.44	0.42	0.10	0.04

Example

***Rule 3: P*(A) = 1 – *P*(not A)**

- It can be written as

  P(not A) = 1 – *P*(A) or P(A)+P(not A) = 1

Blood type	O	A	B	AB
Probability	0.44	0.42	0.10	0.04

What is probability that a randomly selected person does NOT have blood type A?

P(not A) = 1 – P(A) = 1 – 0.42 = 0.58

Note

***Rule 3: P*(A) = 1 – *P*(not A)**

- It can be written as

  P(not A) = 1 – *P*(A) or P(A)+P(not A) = 1

- In some cases, when finding P(A) directly is very complicated, it might be much easier to find P(not A) and then just subtract it from 1 to get the desired P(A).

Odds

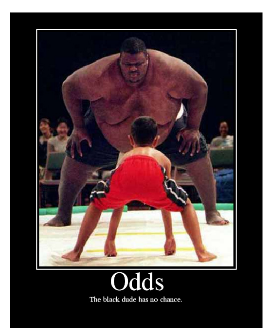
- **Odds against** event A:

  $$\frac{P(\bar{A})}{P(A)} = \frac{P(not\ A)}{P(A)}$$

  expressed as a:b

- **Odds in favor** event A:

  $$\frac{P(A)}{P(\bar{A})} = \frac{P(A)}{P(not\ A)}$$



Example

Event A: rain tomorrow.

The probability of rain tomorrow is 80% → **P(A) = 0.8**

- **What are the odds against the rain tomorrow?**

$$\frac{P(\bar{A})}{P(A)} = \frac{P(not\ A)}{P(A)} = \frac{0.2}{0.8} = \frac{1}{4} = 1{:}4$$

- **What are the odds in favor of rain tomorrow?**

$$\frac{P(A)}{P(\bar{A})} = \frac{P(A)}{P(not\ A)} = \frac{0.8}{0.2} = \frac{4}{1} = 4{:}1$$

Example: lottery

- **The odds in favor of winning a lottery is 1:1250**
- **This means that the probability of winning is**

$$\frac{1}{1251} = 0.0008 = 0.08\%$$

Rule 4

- We are now moving to rule 4 which deals with another situation of frequent interest, finding **P(A *or* B),** the probability of one event ***or*** another occurring.
- In probability "OR" means either one or the other or both, and so,

  P(A or B) = P(event A occurs or event B occurs or both occur)

Examples

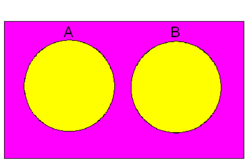
- Consider the following two events:
  - A - a randomly chosen person has blood type A, and
  - B - a randomly chosen person has blood type B.

  Since a person can only have one type of blood flowing through his or her veins, it is impossible for the events A and B to occur together.
- On the other hand...Consider the following two events:
  - A - a randomly chosen person has blood type A
  - B - a randomly chosen person is a woman.

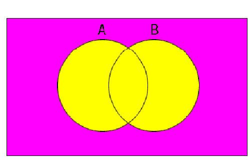
  In this case, it *is possible* for events A and B to occur together.

Disjoint or Mutually Exclusive Events

Definition: Two events that cannot occur at the same time are called ***disjoint*** or ***mutually exclusive***.



A and B are Disjoint



A and B are NOT Disjoint

Decide if the Events are Disjoint

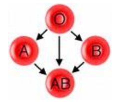
- **Event A**: Randomly select a female worker.

  Event B: Randomly select a worker with a college degree.
- **Event A**: Randomly select a male worker.

  Event B: Randomly select a worker employed part time.
- **Event A**: Randomly select a person between 18 and 24 years old.

  Event B: Randomly select a person between 25 and 34 years old.

Example



- **Rule 4: P(A or B) = P(A) + P(B)**

Blood type	O	A	B	AB
Probability	0.44	0.42	0.10	0.04

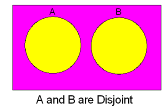
What is the probability that a randomly selected person has either blood type A or B?

Since "blood type A" is disjoint of "blood type B", P(A or B) = P(A) + P(B) = 0.42 + 0.10 = 0.52

More Rules…

- Recall: the **Addition Rule for disjoint events** is

P(A **or** B) = P(A) **+** P(B)

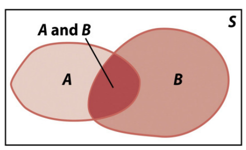


But what rule can we use for NOT disjoint events?

The general addition rule

General addition rule for <u>any</u> two events A and B:

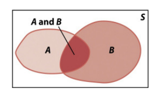
The probability that A occurs, or B occurs, or both events occur is:



P(A **or** B) = P(A) **+** P(B) **–** P(A and B)

The general addition rule: example

What is the probability of randomly drawing either an **ace or a heart** from a pack of 52 playing cards?



There are 4 aces in the pack and 13 hearts.

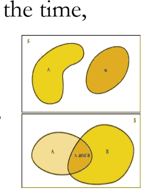
However, one card is both an ace and a heart. Thus:

***P*(ace or heart) = *P*(ace) + *P*(heart) – *P*(ace and heart)**

= 4/52 + 13/52 - 1/52 = 16/52 ≈ 0.3

Note

- The General Addition Rule works ALL the time, for ANY two events
- P(A **or** B) = P(A) **+** P(B) **–** P(A and B)
- Note that if A and B are disjoint events,



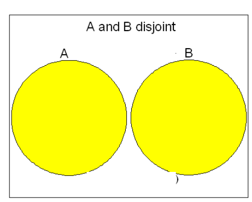
P(A and B)=0, thus

P(A **or** B) = P(A) **+** P(B) **–** $\overbrace{P(\text{A and B})}^{0}$

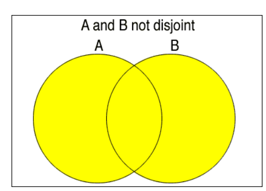
= P(A) **+** P(B)

Which is the Addition Rule for Disjoint Events.

Addition Rules: Summary



P(A or B)=P(A)+P(B) since P(A and B)=0



P(A or B)=P(A)+P(B)-P(A and B)

Probability Rules

1. The probability P(A) for any event A is $0 \leq P(A) \leq 1$.
2. If S is the sample space in a probability model, then P(S)=1.
3. For any event A, P(A does not occur) = 1- P(A).
4. If A and B are disjoint events, P(A or B)=P(A)+P(B).
5. For any two events,

   P(A or B) = P(A)+P(B)-P(A and B).

Probability definition

A correct interpretation of the statement "The probability that a child delivered in a certain hospital is a girl is 0.50" would be which one of the following?

a) Over a long period of time, there will be equal proportions of boys and girls born at that hospital.
b) In the next two births at that hospital, there will be exactly one boy and one girl.
c) To make sure that a couple has two girls and two boys at that hospital, they only need to have four children.
d) A computer simulation of 100 births for that hospital would produce exactly 50 girls and 50 boys.

Probability

From a computer simulation of rolling a fair die ten times, the following data were collected on the showing face:

5 5 1 3 2 1 5 6 5 1

What is a correct conclusion to make about the next ten rolls of the same die?

a) The probability of rolling a 5 is greater than the probability of rolling anything else.
b) Each face has exactly the same probability of being rolled.
c) We will see exactly three faces showing a 1 since it is what we saw in the first experiment.
d) The probability of rolling a 4 is 0, and therefore we will not roll it in the next ten rolls.

Probability models

If a couple has three children, let X represent the number of girls. Does the table below show a correct probability model for X?

X	0	1	2	3
Proportion	0.125	0.375	0.375	0.125

a) No, because there are other values that X could be.
b) No, because it is not possible for X to be equal to 0.
c) Yes, because all combinations of children are represented.
d) Yes, because all probabilities are between 0 and 1 and they sum to 1.

Probability

If a couple has three children, let X represent the number of girls. What is the probability that the couple does NOT have girls for all three children?

X	0	1	2	3
Proportion	0.125	0.375	0.375	0.125

a) 0.125
b) 0.125 + 0.375 = 0.500
c) 1 – 0.125 = 0.825

Probability

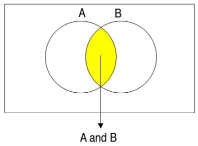
If a couple has three children, let X represent the number of girls. What is the probability that the couple has either one or two boys?

X	0	1	2	3
Proportion	0.125	0.375	0.375	0.125

a) 0.375
b) 0.375 + 0.375 = 0.750
c) 1 - 0.125 = 0.825
d) 0.500

More rules: P(A and B)

- Another situation of frequent interest, finding P(A *and* B), the probability that both events A and B occur.



- P(A and B)= P(event A occurs **and** event B occurs)
- As for the Addition rule, we have two versions for P(A and B).

Independent events

- Two events are **independent** if the probability that one event occurs on any given trial of an experiment is not influenced in any way by the occurrence of the other event.
- Example: toss a coin twice
  - Event A: first toss is a head (H)
  - Event B: second toss is a tail (T)

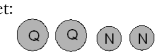
Events A and B are independent. The outcome of the first toss cannot influence the outcome of the second toss.

Example

- Imagine coins spread out so that half were heads up, and half were tails up. Pick a coin at random. The probability that it is heads-up is 0.5. But, if you don't put it back, the probability of picking up another heads-up coin is now less than 0.5. **Without replacement, successive trials are not independent.**
- In this example, the trials are independent only when you put the coin back ("sampling with replacement") each time.

Example

- A woman's pocket contains 2 quarters and 2 nickels. She randomly extracts one of the coins and, after looking at it, **replaces** it before picking a second coin.
- Let Q1 be the event that the first coin is a quarter and Q2 be the event that the second coin is a quarter.
- Are Q1 and Q2 independent events? *YES!* Why?
- Since the first coin that was selected is *replaced*, whether Q1 occurred (i.e., whether the first coin was a quarter) has no effect on the probability that the second coin is a quarter, P(Q2). In either case (whether Q1 occurred or not), when we come to select the second coin, we have in our pocket:



More Examples

- EXAMPLE: Two people are selected at random from all living humans. Let B1 be the event that the first person has blue eyes and B2 be the event that the second person has blue eyes. In this case, since the two were chosen at random, whether the first person has blue eyes has no effect on the likelihood of choosing another blue eyed person, and therefore B1 and B2 are independent. On the other hand.....
- EXAMPLE: A family has 4 children, two of whom are selected at random. Let B1 be the event that the first child has blue eyes, and B2 be the event that the second chosen child has blue eyes. In this case B1 and B2 are not independent since we know that eye color is hereditary, so whether the first child is blue-eyed will increase or decrease the chances that the second child has blue eyes, respectively.

Decide whether the events are independent or dependent

- **Event A:** a salmon swims successfully through a dam

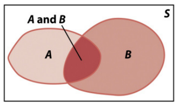
  Event B: another salmon swims successfully through the same dam
- **Event A:** parking beside a fire hydrant on Tuesday

  Event B: getting a parking ticket on the same Tuesday

The Multiplication Rule for **independent events**

If A and B are independent,

$P(A$ **and** $B) = P(A) \cdot P(B)$



The Multiplication Rule for **independent events:** Example

- Roll two dice.
  - Event A: roll a "6" on a red die
  - Event B: roll a "5" on a blue die.
- What is the probability that if you roll both at the same time, you will roll a "6" on the red die **and** a 5 on the blue die? Since the two dice are independent,

P(A and B)=P(A)·P(B)=1/6·1/6 = 1/36

Example

- Recall the blood type example:

Blood type	O	A	B	AB
Probability	0.44	0.42	0.10	0.04

- Two people are selected at random from all living humans. What is the probability that both have blood type O?
- Let O1= "the first has blood type O" and O2= "the second has blood type O"
- We need to find P(O1 and O2)
- Since the two were chosen at random, the blood type of one has no effect on the blood type of the other. Therefore, O1 and O2 are independent and we may apply Rule 5:
- P(O1 and O2) = P(O1)*P(O2) = .44*.44=.1936.

Probability Rules

1. The probability P(A) for any event A is $0 \leq P(A) \leq 1$.
2. If S is the sample space in a probability model, then P(S)=1.
3. For any event A, P(A does not occur) = 1- P(A).
4. If A and B are disjoint events, P(A or B)=P(A)+P(B).
5. General Addition Rule: For any two events, P(A or B) = P(A)+P(B)-P(A and B).
6. If A and B are independent, $P(A$ **and** $B) = P(A) \cdot P(B)$

One more rule

- We need a general rule for P(A and B).
- For this, first we need to learn Conditional probability.
- Notation: P(B|A). This means the probability of event B, given that event A has occurred. Event A represents the information that is given.

Example

- All the students in a certain high school were surveyed, then classified according to gender and whether they had either of their ears pierced:

	Pierced	Not pierced	Total
Male	36	144	180
Female	288	32	320
Total	324	176	500

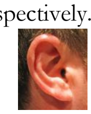
Example

- Suppose a student is selected at random from the school.
- Let ***M*** and ***not M*** denote the events of being male and female, respectively, and ***E*** and ***not E*** denote the events of having ears pierced or not, respectively.


M


not M


E


not E

	Pierced	Not pierced	Total
Male	36	144	180
Female	288	32	320
Total	324	176	500

- What is the probability that the student has one or both ears pierced?
  Since a student is chosen at random from the group of 500 students out of which 324 are pierced, **P(E)=324/500=.648**
- What is the probability that the student is a male?
  Since a student is chosen at random from the group of 500 students out of which 180 are males, **P(M)=180/500=.36**
- What is the probability that the student is male and has ear(s) pierced?
  Since a student is chosen at random from the group of 500 students out of which 36 are males and have their ear(s) pierced, **P(M and E)=36/500=.072**

Conditional Probability

- Now something new:
- ***Given*** **that the student that was chosen is a male, what is the probability that he has one or both ears pierced?**
- We will write "the probability of having one or both ears pierced (E) , given that a student is male (M)" as *P(E|M).*
- We call this probability the ***conditional probability*** of having one or both ears pierced, given that a student is male: it assesses the probability of having pierced ears under the condition of being male.

	Pierced	Not pierced	Total
Male	36	144	180
Female	288	32	320
Total	324	176	500

- The total number of possible outcomes is no longer 500, but has changed to 180. Out of those 180 males, 36 have ear(s) pierced, and thus:

P(E|M)=36/180=0.20.

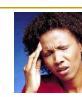
	Pierced	Not pierced	Total
Male	36	144	180
Female	288	32	320
Total	324	176	500

General formula

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

- The above formula holds as long as P(A)>0 since we cannot divide by 0. In other words, we should not seek the probability of an event given that an impossible event has occurred.

Example



- On the "Information for the Patient" label of a certain anti-depressant it is claimed that based on some clinical trials, there is a 14% chance of experiencing sleeping problems known as **insomnia** (denote this event by **I**), there is a 26% chance of experiencing **headache** (denote this event by **H**), and there is a 5% chance of experiencing both side effects (**I and H**).
- Thus, **P(I)=0.14** **P(H)=0.26** **P(I and H)=0.05**

Example $P(I)=0.14 \quad P(H)=0.26 \quad P(I \text{ and } H)=0.05$

- (a) Suppose that the patient experiences insomnia; what is the probability that the patient will also experience headache?
- Since we know (or it is given) that the patient experienced insomnia, we are looking for P(H|I). According to the definition of conditional probability:

  P(H|I)=P(H and I)/P(I)=.05/.14=.357.

- (b) Suppose the drug induces headache in a patient; what is the probability that it also induces insomnia?
- Here, we are given that the patient experienced headache, so we are looking for P(I|H).

  P(I|H)=P(I and H)/P(H)=.05/.26=.1923.

Important!!!

In general,

$$P(A|B) \neq P(B|A)$$

Independence

- Recall: two events A and B are *independent* if one event occurring *does not affect* the probability that the other event occurs.
- Now that we've introduced conditional probability, we can formalize the definition of independence of events and develop four simple ways to check whether two events are independent or not.

Independence Checking

- This example illustrates that one method for checking whether two events are independent is to compare *P(B|A)* and *P(B)*.
- If $P(B|A) = P(B)$ then the two events, A and B, are ***independent***.
- If $P(B|A) \neq P(B)$ then the two events, A and B, are not *independent* (they are ***dependent***).

Similarly, using the same reasoning, we can compare *P(A|B)* and *P(A)*.

Example

- Recall the side effects example. "… there is a 14% chance of experiencing sleeping problems known as insomnia (*I*), there is a 26% chance of experiencing headache (*H*), and there is a 5% chance of experiencing both side effects (*I and H*).
- Thus, P(I)=0.14 P(H)=0.26 P(I and H)=0.05
- **Are the two side effects independent of each other?**

Example

- To check whether the two side effects are independent, let's compare P(H|I) and P(H).
- In the previous part of this lecture, we found that P(H|I)=P(H and I)/P(I)=.05/.14=.357 while P(H)=.26

  P(H|I) ≠ P(H)

  Knowing that a patient experienced insomnia increases the likelihood that he/she will also experience headache from .26 to .357. The conclusion, therefore is that the two side effects are not independent, they are **dependent**.

General Multiplication Rule

- For independent events A and B, we had the rule
  P(A and B)=P(A)*P(B).
- Now, for events A and B that may be dependent, to find the probability of both, we multiply the probability of A by the conditional probability of B, taking into account that A has occurred. Thus, our general multiplication rule is stated as follows:
- ***Rule 7: The General Multiplication Rule:* For any two events A and B, *P(A and B)=P(A)*P(B|A)***

Comments ***P(A and B)=P(A)*P(B|A)***

- This rule is general in the sense that if A and B happen to be *independent*, then P(B|A)=P(B) is, and we're back to Rule 5 - the Multiplication Rule for Independent Events: P(A and B)=P(A)*P(B).
- Recall the definition of conditional probability: *P(B|A)=P(A and B)/P(A)*. Let's isolate P(A and B) by multiplying both sides of the equation by P(A), and we get: *P(A and B)=P(A)*P(B|A)*. That's it....this is the General Multiplication Rule.

Probability Rules

1. The probability P(A) for any event A is $0 \leq P(A) \leq 1$.
2. If S is the sample space in a probability model, then P(S)=1.
3. For any event A, P(A does not occur) = 1- P(A).
4. If A and B are disjoint events, P(A or B)=P(A)+P(B).
5. General Addition Rule: For any two events, P(A or B) = P(A)+P(B)-P(A and B).
6. If A and B are independent, $P(A \text{ and } B) = P(A) \cdot P(B)$
7. General Multiplication Rule: $P(\text{ A and B}) = P(A) \cdot P(B|A)$

Probability Rules

Three students work independently on a homework problem.
The probability that the first student solves the problem is 0.95.
The probability that the second student solves the problem is 0.85.
The probability that the third student solves the problem is 0.80.

What is the probability that all are able to solve the problem?

a) 0.95 + 0.85 + 0.80
b) (0.95) (0.85) (0.80)
c) 1 – 0.95 – 0.85 – 0.80
d) 1 - (0.95) (0.85) (0.80)
e) 0.80

Probability Rules

Three students work independently on a homework problem.
The probability that the first student solves the problem is 0.95.
The probability that the second student solves the problem is 0.85.
The probability that the third student solves the problem is 0.80.

What is the probability that the first student solves the problem and the other two students do not?

a) 0.95 + 0.15 + 0.20
b) (0.95) (0.15) (0.20)
c) 0.95 – 0.15 – 0.20
d) 0.95 – 0.85 – 0.80
e) 0.95

Probability Rules

Three students work independently on a homework problem.
The probability that the first student solves the problem is 0.95.
The probability that the second student solves the problem is 0.85.
The probability that the third student solves the problem is 0.80.

What is the probability that none of the three students solves the problem?

a) 1 - 0.95 – 0.85 – 0.80
b) 0.05 + 0.15 + 0.20
c) 1 – (0.95) (0.85) (0.80)
d) (0.05) (0.15) (0.20)

Probability Rules

Three students work independently on a homework problem.

The probability that the first student solves the problem is 0.95.

The probability that the second student solves the problem is 0.85.

The probability that the third student solves the problem is 0.80.

What is the probability that the first student solves the problem or the second student solves the problem?

a) 0.95 + 0.85 + 0.80
b) (0.95) (0.85)
c) 0.95 + 0.85
d) (0.95) (0.85) (0.20)

Probability Rules

Three students work independently on a homework problem.

The probability that the first student solves the problem is 0.95.

The probability that the second student solves the problem is 0.85.

The probability that the third student solves the problem is 0.80.

What is the probability that at least one of them solves the problem correctly?

a) 0.95 + 0.85 + 0.80
b) (0.95) (0.85) (0.80)
c) 1 – (0.05) (0.15) (0.20)
d) 0.95
e) 0.80

Independence

Chris is taking two classes this semester, English and American History.

The probability that he passes English is 0.50.

The probability that he passes American History is 0.40.

The probability that he passes both English and American History is 0.60.

Are passing English and passing American History independent events?

a) Yes, because the classes are taught by different teachers.
b) Yes, because the classes use different skills.
c) No, because (0.50) (0.40) ≠ 0.60.
d) No, because (0.50) ≠ (0.40) (0.60).

Probability Rules

Chris is taking two classes this semester, English and American History.

The probability that he passes English is 0.50.

The probability that he passes American History is 0.40.

The probability that he passes both English and American History is 0.60.

What is the probability Chris passes either English or American History?

a) 0.50 + 0.40
b) 0.50 + 0.40 – 0.60
c) 0.60 – 0.50 – 0.40
d) 0.50 + 0.40 + 0.60
e) (0.50) (0.40) – 0.60

Conditional probability

At a certain university, 47.0% of the students are female.

Also, 8.5% of the students are married females.

If a student is selected at random, **what is the probability that the student is married, given that the student was female?**

a) 0.085 / 0.47
b) 0.47 / 0.085
c) (0.085) (0.47)
d) 0.085
e) 0.47

Conditional probability

Within the United States, approximately 11.25% of the population is left-handed.

Of the males, 12.6% are left-handed, compared to only 9.9% of the females.

Assume the probability of selecting a male is the same as selecting a female.

If a person is selected at random, **what is the probability that the selected person is a left-handed male?**

a) (0.126) (0.50)
b) (0.126)
c) (0.1125) (0.50)
d) 0.126 / 0.50