Practice Problems for Test 3

Note: these problems only cover CIs and hypothesis testing. You are also responsible for knowing the sampling distribution of the sample means, and the Central Limit Theorem. Review all the exercises we did in class, and the homework problems also.

(Please let me know if I have any mistakes on this review sheet—extra credit)

1. **From a random sample of 36 days in a recent year, the closing stock prices of Hasbro had a <u>mean</u> of \$19.31and standard deviation of \$2.37. Use t* = 1.68 for 90%, and t* = 2.03 for 95% confidence.**

a. Construct the 90% and 95% confidence intervals for the population mean. Interpret the CIs.

<u>90% CI:</u> Conditions: SRS, and n >30.

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}} = 19.31 \pm 1.68 \frac{2.37}{\sqrt{36}} = (18.643, 19.977)$$

Or if you have a TI-83 use STAT→TESTS→8:T-Interval

We are 90% confident that the mean closing stock prices of Hasbro is between \$18.64 and \$19.98.

<u>95% CI:</u> Conditions: SRS, and n >30.

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}} = 19.31 \pm 2.03 \frac{2.37}{\sqrt{36}} = (18.508, 20.112)$$

Or if you have a TI-83 use STAT→TESTS→8:T-Interval

We are 95% confident that the mean closing stock prices of Hasbro is between \$18.51 and \$20.11.

c. Which interval is wider? What are the margins of errors?

The 95% CI is wider. And it is suppose to be wider, because higher confidence level gives wider CI intervals.

Margin of error for 90% confidence:

$$1.68 \frac{2.37}{\sqrt{36}} = 0.66$$

Margin of error for 95% confidence:

$$2.03 \frac{2.37}{\sqrt{36}} = 0.802$$

2. **You randomly select 20 mortgage institutions and determine the current mortgage interest rate at each. The sample mean rate is 6.93% with a sample standard deviation of 0.42%. Find the 99% confidence interval for the population <u>mean</u> mortgage interest rate. Assume the interest rates are approximately normally distributed.**

   Conditions: SRS, population is normally distributed. Both satistfied.

   t* =2.861

   $$\bar{x} \pm t^* \frac{s}{\sqrt{n}} = 6.93 \pm 2.861 \frac{0.42}{\sqrt{20}} = (6.66, 7.20)$$

   We are 99% confident that the mean mortgage interest rate for ALL mortgage institutions is between 6.66% and 7.20%.

3. **A biologist reports a CI of (2.1cm, 3.5cm) when estimating the <u>mean</u> height of a sample of seedlings. Find the sample mean and the margin of error.**

   The point estimator, the sample mean is exactly in the middle of the CI. The margin of error, *m*, is the half of the interval.

   $$\begin{array}{ccccc} & m & & m & \\ (& \text{-------------------------} & | & \text{-------------------------} &) \\ 2.1\text{ cm} & & \bar{x} & & 3.5\text{ cm} \end{array}$$

   Thus, the sample mean is: $\bar{x} = \frac{2.1 + 3.5}{2} = 2.8\ cm$

   And the margin of error is: $m = 2.8\ cm - 2.1\ cm = 0.7\ cm$

4. **A random sample of airfare prices in dollars for a one-way ticket from New York to Houston is given:**

   288, 290, 292, 295, 298, 300, 305, 305, 306, 307, 314, 320, 322, 326, 327

   Assuming that the prices of tickets are normally distributed, find and interpret the 90% CI for the mean price of all tickets from New York to Houston. (The sample mean is $306.34, and the sample standard deviation is $12.97, t* = 1.761)

   Conditions: SRS, normally distributed population: both are satisfied.

   If you have a TI-83, 84, you can enter the list and then use STAT → TESTS → 8: TInterval and highlight Data.

90% CI: $\bar{x} \pm t^* \frac{s}{\sqrt{n}} = 306.34 \pm 1.761 \frac{12.97}{\sqrt{15}} = (300.43, 312.23)$

We are 90% confident that the mean price of ALL airfare prices for a one-way ticket from New York to Houston is between \$300.43 and \$312.23.

5. The following table shown is from a survey of randomly selected 900 U.S. adults.

Who are the more dangerous drivers?	
Teenagers	565
People over 75	297
No opinion	38

a. Construct a 95% CI for the proportion of adults who think that teenagers are the more dangerous drivers. What's the margin of error?

Conditions: SRS,

$$\hat{p} = \frac{x+2}{n+4} = \frac{565+2}{900+4} = 0.627$$

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n+4}} = 0.627 \pm 1.96 \sqrt{\frac{0.627(1-0.627)}{904}} = (0.596, 0.659)$$

We are 95% confident that the proportion of ALL adults who think that teenagers are the more dangerous drivers is between 59.6% and 65.9%.

The margin of error: $1.96 \sqrt{\frac{0.627(1-0.627)}{904}} = 0.0315 \approx 3\%$

b. Construct a 95% CI for the proportion of adults who think that people over 75 are the more dangerous drivers. What is the margin of error?

Conditions: SRS,

$$\hat{p} = \frac{x+2}{n+4} = \frac{297+2}{900+4} = 0.331$$

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n+4}} = 0.331 \pm 1.96 \sqrt{\frac{0.331(1-0.331)}{904}} = (0.298, 0.359)$$

We are 95% confident that the proportion of ALL adults who think that teenagers are the more dangerous drivers is between 29.8% and 35.9%.

The margin of error: $1.96\sqrt{\frac{0.331(1-0.331)}{904}} = 0.0307 \approx 3\%$

6. **You are running a political campaign and wish to estimate, with 99% confidence, the proportion of registered voters who will vote for your candidate. Your estimate must be accurate within 3% of the true population. Find the minimum sample size needed if**
   a. no preliminary estimation is available

   When no preliminary estimation is available, use $\hat{p} = 0.5$

   $$n = \left(\frac{z^*}{m}\right)^2 \hat{p}(1-\hat{p}) = \left(\frac{2.576}{0.03}\right)^2 (0.5)(1-0.5) = 1843.271$$

   Thus, the minimum sample size required is 1844.

b. a preliminary estimation gives $\hat{p} = 0.31$.

   $$n = \left(\frac{z^*}{m}\right)^2 \hat{p}(1-\hat{p}) = \left(\frac{2.576}{0.03}\right)^2 (0.31)(1-0.31) = 1577.102$$

   Thus, the minimum sample size required is 1578.

7. **A used car dealer says that the mean price of a 2002 Ford F-150 Super Cab is \$18,800. You suspect this claim is <u>too high</u>. You find that a random sample of 14 similar vehicles has a mean price of \$18,250 and a standard error of \$1,250. Is there enough evidence to reject the dealer's claim at α=0.05? Assume the population is normally distributed. Is your conclusion the same at the 10% level? Can you reach the same conclusion by using a CI?**

   $H_0: \mu = 18800$

   $H_a: \mu < 18800$

   Conditions: SRS, normal distribution. Both satisfied.

   $$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{1820 - 18800}{\frac{1250}{\sqrt{14}}} = -1.646$$

   The p-value is 0.0618.

Since the p-value is greater than 0.05, the significance level, we cannot reject the null hypothesis. At the 5% level, we don't have enough evidence to reject the claim that the mean price of 2002 Ford F-150 Super Cab is \$18,800.

At the 10% level, since the p-value is less than 10%, we can reject the null hypothesis. We have enough evidence to reject the claim that the mean price of 2002 Ford F-150 Super Cab is $18,800.

No, we can't use a CI to test the claim because it's not a two-tailed test. (We don't have an ≠ in the alternative hypothesis).

8. **A company that makes cola drinks states that the mean caffeine content per one 12-ounce bottle of cola is 40 milligrams. You work as a quality control manager and are asked to test this claim. During your tests, you find that a random sample of 30 12-ounce bottles of cola has a mean caffeine content of 39.2 milligrams with standard deviation of 7.5 milligrams. Assume that the caffeine content is normally distributed. At α = 1% level, can you reject the company's claim? Can you reach the same conclusion by using a CI?**

$H_0: \mu = 40$

$H_a: \mu \neq 40$

Conditions: SRS, the population is normally distributed.

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{39.2 - 40}{\frac{7.5}{\sqrt{30}}} = -0.584$$

p-value: 0.559

Conclusion: since the p-value is about 56%, we have no evidence to reject the null hypothesis. We don't have enough evidence to reject the claim that the mean caffeine level of all 12-ounce bottle of cola is 40 milligrams.

Yes, we can use a 99% confidence interval to test the claims since we have a two-sided test. Use t* = 2.75

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}} = 39.2 \pm 2.75 \frac{7.5}{\sqrt{30}} = (35.43, 42.97)$$

Since the 99% CI does contain the hypothesized value, 40, we cannot reject the null hypothesis at the 1% level.

9. **A scientist estimates that the mean nitrogen dioxide level in West London is 28 parts per billion. You believe that the mean nitrogen dioxide level is higher. So, you determine the nitrogen dioxide levels for 36 randomly selected days. The results (in parts per billion) are listed below. At α = 10%, can you support the scientist's estimate? Can you reach the same conclusion by using a CI?**

27, 29, 53, 31, 16, 47, 22, 17, 13, 46, 99, 15
20, 17, 28, 10, 14, 9, 35, 29, 32, 67, 24, 31,
43, 29, 12, 39, 65, 94, 12, 27, 13, 16, 40, 62

(The sample mean is 32.86, and the sample standard deviation is 22.13)

Hypotheses: $H_0: \mu = 28$
$H_a: \mu > 28$

Random sample,and n > 30.

If you have a Ti-83, 84, you can enter the list and then use STAT → TESTS→ 2: T-test, and highlight Data.

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{32.86 - 28}{\frac{22.12}{\sqrt{36}}} = 1.318$$

p-value: 0.098

Conclusion: since p-value is less than α (0.098 < 10), strictly speaking the result is statistically significant. That is, we can reject the null hypothesis, although it's really a borderline case since 0.098 ≈ 0.10.
The decision is in your hands.

This would be my conclusion: We have little or no evidence against the null hypothesis. We have little or no evidence against the claim that the mean nitrogen dioxide level in West London is 28 parts per billion. Further investigation is necessary.

No, we can't use a CI interval here to test the claim because it's not a two-tailed test (we don't have $\neq$ in the alternative hypothesis.)

10. **A medical researcher claims that 20% of adults in the U.S. are allergic to a medication. You think that less than 20% are allergic to a medication. In a random sample of 100 adults, 15% say they have such an allergy. Test the researcher's claim at α = 1% level. Can you reach the same conclusion using the appropriate confidence interval?**

Hypotheses: $H_0: p = 0.20$
$H_a: p < 0.20$

Conditions: SRS, $np_0 = 100(0.20) = 20 > 10, \quad n(1-p_0) = 100(1-0.20) = 80 > 10$, all satistfied.

Test statistic: $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} = \dfrac{0.15 - 0.20}{\sqrt{\dfrac{0.20(1-0.20)}{100}}} = -1.25$

(You can use your calculator, STAT→TESTS→5:1-PropZTest. Use $x = n\hat{p} = 100(0.15) = 15$)

p-value: 0.1056

Conclusion: Since p-value > 10%, we have no evidence against the null hypothesis. We can't reject it. Based on our sample, we don't have enough evidence to reject the medical researcher's claim that 20% of adults in the U.S. are allergic to a medication.

No, we can't use a CI interval here to test the claim because it's not a two-tailed test (we don't have $\neq$ in the alternative hypothesis.)

11. USA TODAY reports that 5% of U.S. adults have seen an extraterrestrial being. You decide to test this claim and ask a random sample of 250 U.S. adults whether they have ever seen an extraterrestrial being. Of those surveyed, 8% reply yes. At α = 5%, is there enough evidence to reject the claim? Can you reach the same conclusion using the appropriate confidence interval?

Hypotheses: $H_0: p = 0.05$
$H_a: p \neq 0.05$

Conditions: SRS, $np_0 = 250(0.05) = 12.5 > 10, \quad n(1-p_0) = 250(1-0.05) = 237.5 > 10$, all satisfied.

Test statistic: $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} = \dfrac{0.08 - 0.05}{\sqrt{\dfrac{0.05(1-0.05)}{250}}} = 2.176$

(You can use your calculator, STAT→TESTS→5:1-PropZTest. Use $x = n\hat{p} = 250(0.08) = 20$)

p-value: 0.029

(If you want to find the p-value using the z-table, find the value for z = 2.176, and multiply if with 2 because it's a two-tailed test.)

Conclusion: Since p-value < 5%, we can reject the null hypothesis. We have good evidence against the USA TODAY's claim that 5% of U.S. adults have seen an extraterrestrial being.

Yes, we can use a 95% CI interval (the confidence level and the α must add up to 100%) because it's a two-tailed test (we have $\neq$ in our alternative hypothesis).

Using the adjusted sample proportion: $\hat{p} = \dfrac{20+2}{250+4} = 0.087$

The 95% CI: $\hat{p} \pm z^* \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n+4}} = 0.87 \pm 1.96\sqrt{\dfrac{0.87(1-0.87)}{250+4}} = (0.052, 0.121)$

TI-83: A: 1-PropZInt with x = 22, n = 254

Since the claimed value, 0.05 is NOT inside the 95% CI, at the 5% level we can reject the null hypothesis claimed by USA TODAY that 5% of U.S. adults have seen an extraterrestrial being. Thus, we can reach the same conclusion what we got using the hypothesis test.