Review for Test 2

Chapters 4, 5 and 6

1. You roll a fair six-sided die. Find the probability of each event:

a. Event A: rolling a 3
1/6

b. Event B: rolling a 7
0

c. Event C: rolling a number less than 5
4/6

d. Event D: rolling an odd number
3/6

e. Event E: rolling a 1 or a 6
2/6

f. Event F: rolling a number that is at least 4
3/6

2. You roll two fair six-sided dice. Find the probability for each event:

a. Event A: the difference of the two number is 3
6/36

b. Event B: the difference of the two numbers is 0
6/36

c. Event C: the sum of the two numbers is less than 6
10/36

d. Event D: the sum of the two numbers is an even number
18/36

e. Event E: the sum of the two numbers is not a 5
32/36

f. Event F: the sum of the two numbers is at least 4
33/36

g. Event G: the sum is 4 or getting doubles
3/36+6/36-1/36 = 8/36

h. Event H: the sum is 7 or getting doubles
6/36+6/36 = 12/36

3. Identify the sample space for each probability experiment:

a. guessing the last digit in a telephone number
S = {0,1,2,3,4,5,6,7,8,9}

b. determining a person's blood type (A, B, AB, O) and Rh-factor (positive, negative)
S = {A+, A-, B+,B-, AB+, AB-, O+, O-}

c. the gender of three children
S = {GGG, GGB, GBG, BGG, GBB, BGB, BBG, BBB}

4. The M&Ms milk chocolate candies have six colors: brown, blue, red, orange, yellow and green. The distribution of the colors are shown in the table below:

Color	brown	blue	red	orange	yellow	green
Probability	13%	24%	13%	20%	14%	

a. What is the probability that a randomly selected candy is green? Explain.
16% because the probabilities must add up to 1.
b. What is the probability that a randomly selected candy is either red or orange?
13%+20% = 33%
c. What is the probability that a randomly selected candy is not blue?
1-0.24 = 0.76 = 76%
d. What is the probability that a randomly selected candy is neither brown nor green?
24% + 13% +20% +14% =71%

5. In a local school, 40% carry a backpack, and 50% carry a wallet. 10% of the students carry both a backpack and a wallet. If a student is selected at random, find the probability that the student carries a backpack or a wallet.

P(B or W) =P(B) +P(W) –P(B and W) =0.4 + 0.5 – 0.1 =0.8

6. What is the probability that three randomly selected persons have a birthday on the 1st of any months?

(12/365)*(12/365) = 0.001

7. Are these pairs of events mutually exclusive (disjoint)?

a. has ridden a roller coaster; has ridden a Ferris wheel
No, they are not disjoint; it is possible that someone has ridden a roller coaster and a Ferris wheel
b. owns a classical music CD; owns a jazz CD
No, they are not disjoint; it is possible that someone owns a classical music CD and a jazz CD
c. is a senior; is a junior
These events are disjoint. Someone cannot be a senior and a junior.
d. has brown hair; has brown eyes
No, they are not disjoint; it is possible that someone has brown hair and brown eyes.
e. is left-handed; is right-handed
This is arguable, but I would say these events are disjoint; someone cannot be left-handed and right-handed. But you can argue about this, because there are some people who have no hand preference. In that case they are not disjoint events.
f. has shoulder-length hair; is male
No, they are not disjoint; it is possible that someone has shoulder-length hair and is a male.
g. rolling doubles with a pair of dice; rolling a sum of 8
No, they are not disjoint. It is possible to roll doubles with a pair of dice and the sum of the two numbers is 8. (that would be 4 and 4—double and the sum is 8)
h. rolling a 3 on one die; rolling a sum of 10

These are disjoint. With a regular die it is impossible to roll a 3 with one die and in the same time the sum of the two numbers is 10. (that would require the second die to show a 7, which is impossible with a regular 6-sided die)

8. Decide whether the events are independent or dependent:

a. a salmon swims successfully through a dam (A) and another salmon swims successfully through the same dam (B)
These events are independent. The second salmon's success doesn't depend on the first salmon's success.

b. tossing a coin and getting a head (A), and then rolling a six-sided die and obtaining a 6 (B)
These events are independent. The die's outcome doesn't depend on the outcome of the coin toss.

c. driving over 85 miles per hour (A), and then getting in a car accident (B)
These events are dependent. You have a higher chance to get in a car accident if you drive over 85 miles per hour.

9. The probability that a salmon swims successfully through a dam is 0.85.

a. find the probability that two salmons successfully swims through the dam
Since the salmons' successes are independent of each other, the probability is (0.85)(0.85) = 0.7225

b. find the probability that three salmons successfully swims through the dam
(0.85)(0.85)(0.85)=0.6141

c. find the probability that none of the three salmons is successful
(1-0.85)(1-0.85)(1-0.85) = 0.0034

10. The table shows the estimated number (in thousands) of earned degrees conferred in the year of 2000 by level and gender.

		Gender		
		Male	Female	**Total**
Level of degree	Associate	208	323	**531**
	Bachelor	502	659	**1161**
	Master	187	227	**414**
	Doctor	27	19	**46**
	Total	**924**	**1228**	**2152**

A person who earned a degree in 2000 is randomly selected. Find the probability of selecting someone who

a. earned a bachelor's degree
1161/2152

b. earned a master's degree
414/2152

c. earned a master or doctorate degree.
(414+46)/2152

d. earned a bachelor or associate degree.
(1161+531)/2152

e. earned an associate degree and the person is a female.
323/2152

f. earned a doctorate degree and the person is a male.
27/2152

g. earned a doctorate degree given that the person is a male.
27/924

h. is a female given that the person earned a master degree.
227/414

i. is a male given that the person earned a doctorate degree.
27/46

j. earned a bachelor degree given that the person is a female.
659/1228

11. A company that makes cartons finds

- The probability of producing a carton with a puncture is 0.05
- The probability that a carton has a smashed corner is 0.08.
- The probability that a carton has a puncture and has a smashed corner is 0.004.

a. What do you think: Are the events "selecting a carton with a puncture" and "selecting a carton with smashed corner" independent? Explain.
Having a smashed corner should not depend on whether the carton had a puncture or not, so these events should be independent.
One way to check it using the given probabilities:
If P(A)P(B) = P(A and B), then the events are independent.
Here we have : (0.05)(0.08) which is 0.004, which is P(A and B), so the events are independent.

b. If a quality inspector randomly selects a carton, find the probability that the carton has a puncture or has a smashed corner.
P(A or B) = P(A)+P(B)-P(A and B) = 0.05 + 0.08 – 0.004 = 0.126

12. A doctor gives a patient a 60% chance of surviving bypass surgery after a heart attack. If the patient survives the surgery, he has a 50% chance that the heart damage will heal. Find the probability that the patient survives and the heart damage heals.
Let BS be the event that the patient survives bypass surgery.
Let H be the event that the heart damage will heal.
Then P(BS) = 0.60, and also we have a conditional probability: GIVEN that the patient survives, the probability that the heart damage will heal is 0.5, that is P(H|BS) = 0.5
We want to know P(BS and H).
Using the formula of the conditional probability:
P(H and BS) = P(H|BS)·P(BS) = (0.6)(0.5) = 0.3

That is the probability that the patient survives and the heart damage heals is 30%.

13. Forty-three race cars started the 2004 Daytona 500. How many ways can the cars finish first, second, and third?
Since order DOES matter for the medals, it's permutation. ${}_{43}P_3 = 74046$

14. How many ways can you arrange the letters of MATHEMATICS?

$$\frac{11!}{2!2!2!} = 6652800$$

15. A state's department of transportation plans to develop a new section of interstate highway and receives 16 bids for the project. The state plans to hire four of the bidding companies. How many ways can the state hire the four companies?
Since order does NOT matter here, it's combination. ${}_{16}C_4 = 1820$

16. A survey asked a sample of people how many vehicles each owns. The random variable X represents the number of vehicles owned.

x	0	1	2	3	4	5
P(x)	0.005	0.330	0.525	0.102	0.029	0.009

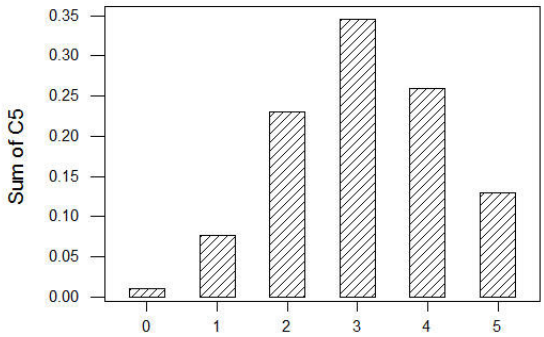
a. Verify that the distribution is a probability distribution.
It is. All the probabilities are between 0 and 1, and the probabilities add up to 1.
b. Find the mean of the distribution.
Mean: 1.852
c. Find the variance and the standard deviation of the distribution.
Standard deviation: 0.79

17. 60% adults say that they have trouble sleeping at night. You randomly select five adults and ask each if he or she has trouble sleeping at night.

a. Decide whether the experiment is a binomial experiment.
It is. The number of trials is fixed (n=5), the probability for each is the same (p=0.6), the trials are independent, and there are only two outcomes, Success: trouble sleeping, and Failure: no trouble sleeping
b. Construct a binomial distribution.

X	P(X)
0	0.01024
1	0.0768
2	0.2304
3	0.3456
4	0.2592
5	0.1296

c. Graph the distribution.



d. Find the mean, variance and standard deviation of the distribution.

Mean: np = 5(0.6) = 3

Standard deviation: $\sigma = \sqrt{np(1-p)} = \sqrt{5(0.6)(1-0.6)} = 1.095$

e. Find the probability that exactly one of the five adults says that he or she has trouble sleeping.
0.0768

f. Find the probability that none of them will say that they have trouble sleeping.
0.01024

g. Find the probability that at least 4 of them say that they have trouble sleeping.
0.2592 + 0.1296 = 0.3888

18. The number of hours per day adults in the U.S. spend on home computers is normally distributed, with a mean of 5 hours and a standard deviation of 1.5 hours. An adult in the U.S. is randomly selected.

a. Find the probability that the hours spent on the home computer by a randomly selected adult is less than 2.5 hours per day.
0.0478

b. Find the probability that the hours spent on the home computer by a randomly selected adult is more than 9 hours per day.
0.0038

c. Find the probability that the hours spent on the home computer by a randomly selected adult is between 3 and 7 hours.
0.8176

19. The annual per capita use of breakfast cereal (in pounds) in the U.S. can be approximated by a normal distribution with mean 16.9 lb, and standard deviation 2.5 lb.

a. What is the smallest annual per capita consumption of breakfast cereal that can be in the top 25% of consumption?
18.59 lbs or more

b. What is the largest annual per capita consumption of breakfast cereal that can be in the bottom 15% of consumption?
14.31 lbs or less

21. The prices for sound-system receivers are normally distributed with mean $625 and a standard deviation of $150.

a. What is the probability that a randomly selected receiver costs less than $610?
0.4602

b. You randomly select 10 receivers. What is the probability that their mean cost is more than $700?
0.0569

22. During a certain week the mean price of gasoline (87) in CA was $3.19 per gallon with standard deviation of $0.08.

a. What is the probability that a randomly selected gas station in CA has gasoline (87) price less than $3.10?
We can't answer to this one. We don't know anything about the population distribution.

b. What is the probability that the mean gasoline (87) price of 18 randomly selected gas stations is more than $3.25?
We can't answer to this one. We don't know anything about the population distribution, and 18 is not big enough to use the CLT.

c. What is the probability that the mean gasoline (87) price of 38 randomly selected gas stations is between $3.20 and $3.22?
Now we can calculate the probability because n >30. We can find the two z-scores with mean 3.19 and s.d. = $\frac{0.08}{\sqrt{38}} = 0.013$
Answer: 0.2104

23. According to Harper's magazine, in the U.S. the time children spend watching television per year follows a normal distribution with mean of 1500 hours and a standard deviation of 250 hours.

a. What percent of children watch television for less than 1200 hours per year? Draw the distribution, shade in the area that represents the percentage, and find its value.

0.115

b. Solve the appropriate equation and fill in the blank:
10% of all children watch more than _1820.4____ hours of television per year.

c. Imagine that all possible random samples of size 25 are taken from the population of U.S. children, and then the means from each sample are graphed to form the sampling distribution of sample means. Using the Central Limit Theorem determine the shape, center, and standard deviation of the distribution, and draw the sampling distribution showing the mean and the standard deviation.

Shape: normal since the population distribution is normal
Mean: same as the population mean = 1500 hours
S.D. = $\frac{250}{\sqrt{25}} = 50$

d. What's the probability that the mean number of hours per year that 25 randomly selected children watch television is more than 1600 hours?
0.0228

e. What's the probability that the mean number of hours per year that 25 randomly selected children watch television is less than 1390 hours?
0.0139