## HOMEWORK 16

1. For each of the following situations, define the parameter and set up appropriate null and alternative hypotheses about it. The parameter has been defined for you in the first problem:
a. Under normal conditions, $64 \%$ of the seeds of the rare plant Botanica statistica germinate.

Scientists at an agricultural research station believe that application of a certain vitamin formulation may increase the germination rate.
$p=$ the proportion of all seeds of Botanica statistica that will germinate if the vitamin formulation is applied
$H_{0}: \mathrm{p}=0.64 \quad H_{a}: \quad \mathrm{p}>0.64$
b. A government agency reports that the proportion of automobiles that come off of assembly lines without significant manufacturing flaws is $96 \%$. A consumer organization suspects that the actual figure is lower than this.
$p=$ the proportion of all automobiles that come off of assembly lines without significant manufacturing flaws.
$H_{0}: \mathrm{p}=0.96 \quad H_{a}: \quad \mathrm{p}<0.96$
c. Nationally, $8 \%$ of homeowners get their house painted in any given year. A random sample of California homeowners is taken to see if the percentage is different from the national figure.
$p=$ the proportion of homeowners who get their house painted in any given year.
$H_{0}: \mathrm{p}=0.08 \quad H_{a}: \quad \mathrm{p} \neq 0.08$
2. A consumer advocacy agency plans to study a new gasoline additive that is being advertised as increasing a car's gas mileage. They plan to collect data from a sample of cars and do a hypothesis test. Either the product increases gas mileage or it doesn't. Which of these two possibilities is the null hypothesis, and which is the alternative? Justify your answer.

The null hypothesis is always the "nothing special is going on" statement, so H0 should be the statement that the product doesn't increase gas mileage.
On the other hand, the alternative hypothesis is the challenging claim, which in this case is that the product increases gas mileage.
3. A medical research team wants to test whether $90 \%$ of the children in a certain African village have been vaccinated against diphtheria. They plan to sample 75 children. Is this a large enough sample to fulfill the conditions for the sampling distribution to be normal?

Need to check two conditions: $n p_{0}$ must be greater than or equal to 10 , and $n\left(1-p_{0}\right)$ must be greater or equal to 10 .

Here, $\mathrm{np}_{0}=75(0.90)=67.5>10$ and $n\left(1-\mathrm{p}_{0}\right)=75(0.1)=7.5 \quad$ this is less than 10 , so this condition is not satisfied.

Therefore, the sample is not large enough to fulfill the conditions for the sampling distribution to be normal.
4. In a recent year, $11 \%$ of college students have applied for study abroad. A survey was conducted to see if that proportion changed this year. An appropriate $95 \%$ confidence interval for the proportion of all college students nationwide who have applied for study abroad has been calculated as $(0.107,0.145)$ based on a random sample of 1200 college students. The appropriate hypothesis test yielded a p-value of 0.08 .
a. State the null and the alternative hypotheses.

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\mathrm{H}_{0}: \mathrm{p}=0.11 \quad \mathrm{H}_{\mathrm{a}}: \mathrm{p} \neq 0.11
$$

b. Check the necessary conditions.
$\mathrm{np}_{0}=1200(0.11)=132>10 \quad \mathrm{n}\left(1-\mathrm{p}_{0}\right)=1200(1-0.11)=1068>10$
c. Explain carefully what the p-value means in this context.

If really $11 \%$ of college students have applied for study abroad, the probability of getting a sample proportion of $12.6 \%$ or more extreme just by chance is 0.08 .
( $12.6 \%$ comes from the confidence interval. The sample proportion is in the middle, that is $(0.107+0.145) / 2=0.126$.
d. Based only on the confidence interval, what would be your conclusion?

Since the claimed value, 0.11 is inside the confidence interval, it's a plausible value. Thus, we can't reject it.
e. Based only on the p-value, what would be your conclusion?

Since the p-value is greater than $5 \%$, the significance level, we cannot reject the null hypothesis. That is, we cannot reject the claim the proportion of college students who applied for study abroad is $11 \%$.
f. Do your conclusions in part d and e agree?

Yes, the conclusions are the same.
g. Pretend you work for an education magazine, and you need to write a report about the results. What would you write in your article?

Based on our sample results with 1200 college students, we are fairly confident that the proportion of college students who have applied for study abroad has not changed from $11 \%$.

Actually, we are $95 \%$ confident that the proportion of such students is somewhere between $10.7 \%$ and $14.5 \%$, although $11 \%$ is almost on the edge of the interval. After conducting an appropriate hypothesis testing, we can conclude that we have some evidence against the claim that the proportion of college student who have applied for study abroad is still $11 \%$, but the evidence wasn't strong enough to reject this claim.

