HOMEWORK 10 Due: next class 3/8

1. Suppose your Statistics professor reports test grades as z-scores, and you got a score of 2.20 on the midterm exam. Are you happy? Write a sentence explaining what your score means.

   YES! That means I have very high scores. My scores are 2.2 standard deviations above the mean scores of the class. I am in the top 2% of the class. (If you look at the z-table, you would see that 98.61% of the class got points below my scores. So I am actually in the top 1.5%!)

2. A town's January high temperatures average 36˚F with a standard deviation of 10˚F, while in July the mean high temperature is 74˚F and the standard deviation is 8˚F. In which month is it more unusual to have a day with a high temperature of 55˚F? Explain.

   We need to compare the z-scores:

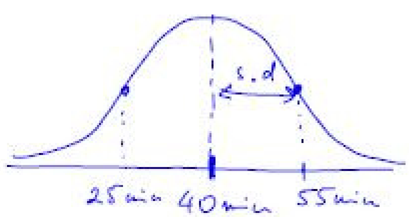
   For January: $z = \frac{x-\mu}{\sigma} = \frac{55-36}{10} = 1.9$

   For July: $z = \frac{x-\mu}{\sigma} = \frac{55-74}{8} = -2.375$

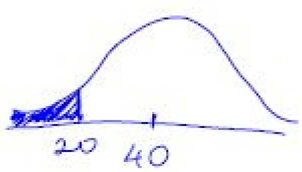
   This means that 55˚F is 1.9 standard deviations above the mean of January high temperatures, and 55˚F is about 2.4 standard deviations below the mean of July's high temperatures. Thus, the 55˚F is more unusual in July.

3. The time that it takes me to make dinner is approximately normal with mean 40 minutes and standard deviation 15 minutes.

   a. *Carefully* draw a normal distribution curve on which this mean and standard deviation are correctly indicated.

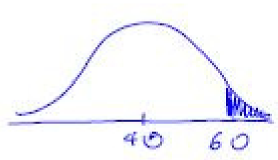
   b. What is the probability that I can make dinner in less than 20 minutes?

   $$z = \frac{x-\mu}{\sigma} = \frac{20-40}{15} = -1.34$$

   You can look up this z-score in the z-table, or using your calculator: normalcdf(-99999,-1.34).
   Either of them will give you the answer about 0.09.
   That it, the probability that I can make dinner in less than 20 minutes is about 9%.

c. What is the chance that I spend more than an hour in the kitchen preparing dinner?

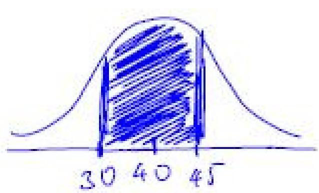


1 hour = 60 min

$$z = \frac{x - \mu}{\sigma} = \frac{60 - 40}{15} = 1.34$$

You can look up this z-score in the z-table (but then you need to subtract the probability from 1), or using your calculator: normalcdf(1.34,999999). Either of them will give you the answer about 0.09.
That it, the probability that I spend more than one hour in the kitchen preparing dinner is about 9% again.

d. How often will I spend between 30 and 45 minutes at the stove making dinner?



$$z = \frac{x - \mu}{\sigma} = \frac{30 - 40}{15} = -0.67$$
$$z = \frac{x - \mu}{\sigma} = \frac{45 - 40}{15} = 0.33$$

You can look up these z-scores in the z-table (then subtract the smaller one from the greater one), or using your calculator: normalcdf(-0.67,0.33). Either of them will give you the answer about 0.38.
That it, the probability that I spend between 30 and 45 minutes at the stove making dinner is about 38% again.

4. John Beale of Stanford, CA, recorded the speeds of cars driving past his house, where the speed limit read 20 mph. Suppose that these recordings follow an approximately normal distribution. The mean of 100 readings was 23.84 mph, with a standard deviation of 3.56 mph.

a. How many standard deviations from the mean would a car going under the speed limit be?

$$z = \frac{x - \mu}{\sigma} = \frac{20 - 23.84}{3.56} = -1.08$$

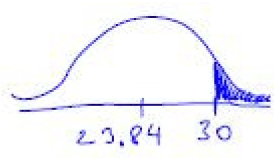
Thus, about 1.08 standard deviations below the mean would a car going under the speed limit be..

b. Which would be more unusual, a car traveling 34 mph or one going 10 mph?

$$z = \frac{x - \mu}{\sigma} = \frac{34 - 23.84}{3.56} = 2.85$$
$$z = \frac{x - \mu}{\sigma} = \frac{10 - 23.84}{3.56} = -3.89$$

Since the z-score of 10 mph is more extreme (farther from zero), going with 10mph is more unusual.

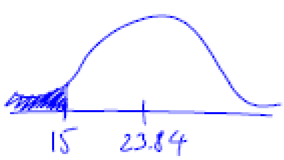
c. What is the probability that a car goes with a speed of 30 mph or more?



$$z = \frac{x - \mu}{\sigma} = \frac{30 - 23.84}{3.56} = 1.73$$

You can look up this z-score in the z-table (but then you need to subtract the probability from 1), or using your calculator: normalcdf(1.73,999999). Either of them will give you the answer about 0.042.
That it, the probability that a car goes with a speed of 30 mph or more is about 4.2%.

d. What is the probability that a car goes with a speed of 15 mph or less?

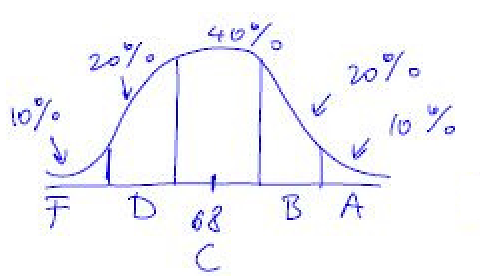


$$z = \frac{x - \mu}{\sigma} = \frac{15 - 23.84}{3.56} = -2.48$$

You can look up this z-score in the z-table, or using your calculator: normalcdf(-99999,-2.48).
Either of them will give you the answer about 0.0066.
That it, the probability that a car goes with a speed of 15 mph or less is about 0.66%.

5. In a large section of a statistics class, the points for the final exam are approximately normally distributed with a mean of 68 and a standard deviation of 9. Grades are assigned according to the following rules:
   - The top 10% receive As
   - The next 20% receive Bs
   - The middle 40% receive Cs
   - The next 20% receive Ds
   - The bottom 10% receive Fs

   Find the lowest score on the final exam that would qualify a student for an A, a B, a C, and a D.



The cutoff value between F and D: you can look up 10% = 0.01 in the z-table (INSIDE the table) and read back the z-score. Or you can use the calculator: DISTR→3:invNorm(0.1). Either way, you should get the z-score of -1.28. Then, to find the actual score we are looking for use the formula:

$x = z \cdot \sigma + \mu = -1.28 \cdot 9 + 68 = 58.47$

Thus, students who got 58.47 or less points on the final received an F.

The cutoff value between D and C: you can look up 30% = 0.03 in the z-table (INSIDE the table) and read back the z-score. Or you can use the calculator: DISTR→3:invNorm(0.3). Either way, you should get the z-score of -0.524. Then, to find the actual score we are looking for use the formula:

$x = z \cdot \sigma + \mu = -0.524 \cdot 9 + 68 = 63.28$

Thus, students who got points between 58.47 and 63.28 got a D.

The cutoff value between C and B: you can look up 70% = 0.07 in the z-table (INSIDE the table) and read back the z-score. Or you can use the calculator: DISTR→3:invNorm(0.7). Either way, you should get the z-score of 0.524. Then, to find the actual score we are looking for use the formula:

$x = z \cdot \sigma + \mu = 0.524 \cdot 9 + 68 = 72.72$

Thus, students who got points between 63.28 and 72.72 got a C.

The cutoff value between B and A: you can look up 90% = 0.09 in the z-table (INSIDE the table) and read back the z-score. Or you can use the calculator: DISTR→3:invNorm(0.9). Either way, you should get the z-score of 1.28. Then, to find the actual score we are looking for use the formula:

$x = z \cdot \sigma + \mu = 1.28 \cdot 9 + 68 = 79.52$

Thus, students who got points between 72.72 and 79.52 got a B, and students who got more than 79.52 points received an A.