Random Variables and Probability Distributions Worksheet

The mean and the standard deviation of a discrete probability distribution are found by using these formulas:

$$Mean: \mu = \sum xP(x)$$

$$Standard\ deviation: \sigma = \sqrt{\sum(x-\mu)^2 \cdot P(x)} = \sqrt{\sum(x^2 \cdot P(x)) - \mu^2}$$

1. **Discrete or Continuous Random Variables?**
a. The time it takes a student selected at random to register for the fall semester
b. The number or bad checks drawn on Upright Bank on a day selected at random
c. The amount of gasoline needed to drive your car 200 miles
d. The number of traffic fatalities per year in the state of Florida
e. The distance a golf ball travels after being hit with a driver
f. The number of ships in Pearl Harbor on any given day
g. Your weight before breakfast each morning

2. **Consider each distribution. Determine if it is a valid probability distribution or not, and explain your answer.**

a.

x	0	1	2
P(x)	0.25	0.60	0.15

b.

x	0	1	2
P(x)	0.25	0.60	0.20

3. USA Today reported that approximately 25% of all state prison inmates released on parole become repeat offenders while on parole. Suppose the parole board is examining five prisoners up for parole. Let x = number of prisoners out of five parole who become repeat offenders, and their corresponding probabilities.

X	0	1	2	3	4	5
P(x)	0.237	0.369	0.264	0.088	0.015	0.001

a. What is the probability that one or more of the five parolees will be repeat offenders? How does this number relate to the probability that none of the parolees will be repeat offenders?
b. Find the probability that two or more of the five parolees will be repeat offenders.
c. Find the probability that two or less of the five parolees will be repeat offenders.
d. Compute the mean number of repeat offenders out of five.
e. Compute the standard deviation of the number of repeat offenders out of five.

A SPECIAL Discrete Probability Distribution: <u>Binomial Distributions</u>

Conditions:
- Fixed number of trials (n is fixed)
- The trials are independent
- The probability of success is the same for each trial
- There are only two outcomes: Success or Failure

In a binomial experiment the probability of exactly x successes in *n* trials is

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x} = \frac{n!}{(n-x)!x!} p^x (1-p)^{n-1}$$

Mean: μ = np
Standard deviation: $\sigma = \sqrt{np(1-p)}$

Problems:

1. **Determine in which of the following situations a binomial distribution can be applied. If so, state and graph the distribution of X, and find the mean and standard deviation of X. If not, state which of the four conditions to satisfy the binomial distribution requirements has been violated.**

   (a) Linda is interested in toilet paper pulling preferences. She takes a simple random sample of 5 people and asks each whether they always pull from the top or not. The probability that a person pulls from the top is 0.53, and X= the number of people who pull from the top.

   (b) I roll a fair, 6-sided die until I get a two. X is the number of rolls it takes before I obtain a roll of two.

   (c) You have a bad containing 4 red chips and 6 white chips and you draw 4 chips. Let random variable Y be the number of red chips drawn from the bag out of 4 draws without replacement.

2. **From the information in 1 (a), answer the following:**

   (a) Find the probability that the number of people who pull from the top is:
   i. Five: P(X = 5) =

   ii. More than three P(X > 3) =

   iii. Between 1 and 3, not including 3: P(1 $\leq$ X < 3) =

   iv. Less than 2: P (X < 2) =

3. The Telektronic Company purchases large shipments of fluorescent bulbs and uses this acceptance sampling plan: Randomly select and test 24 bulbs, then accept the whole batch if there is only one or none that does not work. If a particular shipment of thousands of bulbs actually has a 4% rate of defects, what is the probability that the whole shipment will be accepted?

(a) What does the random variable X represent?

(b) Does X have a binomial distribution? Check the conditions.

(c) What is the probability that the whole shipment will be accepted? (i.e. What is the probability that only one or none of the bulbs do not work?)What is the probability that the whole shipment will not be accepted? (i.e. What is the probability that two or more bulbs will not work?) HINT: Take one minus the probability in (c).

4. Nine percent of all men cannot distinguish between the colors red and green. This is the type of color blindness that causes problems with traffic signals. If six men are randomly selected for a study of traffic signal perceptions:

(a) Determine if X = the number of men that cannot distinguish between red and green is a binomial random variable (check the conditions for the binomial setting).

(b) What is the distribution of X?

(c) Find the probability that exactly two of the six men cannot distinguish between red and green.

(d) What are the mean and standard deviation of the random variable X?