V1

1	2	3	4	5	Σ

Last Name:________________

First Name:________________

ID:________________ Section:____

Math 481a Midterm #1. March 15, 2007

Attention! Please, note that this is the closed book test. You are allowed to use a graphing calculator (but not a palm). In multiple choice, guessing is not penalized. In other problems, please, show all important steps in you solution.

1a. (5pt) The Secant method is most closely related to

- ☐ the bisection method;
- ☐ the Newton's method;
- ☐ the Muller's method;
- ☐ the tangent method;
- ☐ the Horner's method.

1b. (5pt) In the proof of Theorem 2.3 about the convergence of fixed-point iteration:

Theorem 2.3. Let $g \in C[a,b]$ be such that $g(x) \in [a,b]$, for all x in $[a,b]$. Suppose, in addition, that $g'(x)$ exists on (a,b) and that a constant $0 < k < 1$ exists with

$$|g'(x)| \leq k, \quad \text{for all} \quad x \in (a,b).$$

Then, for any number p_0 in $[a,b]$, the sequence defined by

$$p_n = g(p_{n-1}), \quad n \geq 1,$$

converges to the unique fixed point p in $[a,b]$.

the Mean Value Theorem is used to prove that

- ☐ $g(x)$ is continuous;
- ☐ $|p_n - p| = k|p_{n-1} - p|$;
- ☐ $g'(x)$ is continuous;
- ☐ $p_n = g(p_{n-1})$ holds;
- ☐ $|p_n - p| = |g'(\xi_n)||p_{n-1} - p|$, for some $\xi_n \in (a,b)$.

1c. (5pt) In the proof of Theorem 2.5 (convergence of Newton's method):

Theorem 2.5: Let $f(x) \in C^2[a,b]$, if $p \in [a,b]$ exists such that $f(p) = 0$, and $f'(p) \neq 0$, then there exists a $\delta > 0$ such that Newton's method generates a sequence $\{p_n\}_n^\infty$ converging to p for any initial approximation $p_0 \in [p-\delta, p+\delta]$.

the inequality $|g'(x)| \leq k \leq 1$, for all $x \in [p-\delta, p+\delta]$, where δ is some positive number and $g(x) = x - f(x)/f'(x)$ is derived from the assumptions

- ☐ $f'(p) = 0$;
- ☐ $f(p) = 0$;
- ☐ $f(p) = 0$, and $f'(p) \neq 0$;
- ☐ $f(p) = 0$, and $f'(p) = 0$;
- ☐ $f'(p) \neq 0$;

1d. (5pt) The following fact is known about the sequence of approximations $\{p_n\}_{n=1}^\infty$:

$$\lim_{n\to\infty} \frac{|p_{n+1} - p|}{|p_n - p|} = 0.$$

It is correct to say that

- ☐ p_n is divergent;
- ☐ p_n is convergent with the order of convergence bigger than 1;
- ☐ p_n is quadratically convergent;
- ☐ p_n is linearly convergent;
- ☐ p_n converges to zero.

2. (20pt) The Maclaurin series is used to approximate the value of $\sin(x)$ on the interval $[-1, 1]$:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

What is the number of terms that need to be summed in

$$S(x) = \sum_{k=0}^{n} (-1)^k \frac{x^{2k+1}}{(2k+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

to ensure that the error of approximation $|\sin(x) - S(x)|$ is within 10^{-3}.

3. a) (13pt) Write the definition of Newton's method as applied the root finding problem $f(x) = 0$, where $f(x) = 2x^3 + x - 1$. Use the initial approximation $p_0 = 0$.

b) (7pt) If Newton's method is viewed as the functional iteration scheme $p_n = g(p_{n-1})$, what is the function $g(x)$?

4. (20pt) Formulate Horner's method to compute the value of a polynomial $P(x)$ of degree n at point x_0.

5. (10pt) Write the definition of the n-th Lagrange interpolating polynomial

(10pt) Using the nodes $x_0 = 0$, $x_1 = \pi/2$, $x_2 = \pi$, find the second Lagrange interpolating polynomial for $f(x) = \sin(x)$.