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Last Name:________________

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Math 481a Midterm #1. March 16, 2005

Attention! Please, note that this is the closed book test. You are allowed to use a graphing calculator. In multiple choice, guessing is not penalized. In other problems, please, show all important steps in you solution.

1a. (5pt) If one is solving the root problem $f(x) = 0$ by the bisection method, and the initial interval $[a, b]$ contains two roots, then

- ☐ the bisection method does not apply;
- ☐ the bisection section will find only one root;
- ☐ the bisection method will find the root which is closer to the middle of $[a, b]$;
- ☐ the bisection method will find the root that is farther from the middle of $[a, b]$;
- ☐ the bisection method will find both roots.

1b. (5pt) Which from the following is the MINIMUM requirements to guarantee that the fixed point equation $g(x) = x$ has a solution on $[a, b]$

- ☐ $g(x)$ is continuous on $[a, b]$;
- ☐ $g(x)$ is continuous on $[a, b]$, and the values of $g(x)$ belong to $[a, b]$ ($g(x)$ maps $[a, b]$ into itself);
- ☐ $g(x)$ is continuous on $[a, b]$, and the values of $g(x)$ belong to $[a, b]$ ($g(x)$ maps $[a, b]$ into itself), and $g'(x)$ exists on (a, b);
- ☐ $g(x)$ is continuous on $[a, b]$, and the values of $g(x)$ belong to $[a, b]$ ($g(x)$ maps $[a, b]$ into itself), and $g'(x)$ exists on (a, b), and a constant k, $0 < k < 1$ exists such that $|g'(x)| \leq k$ for all $x \in (a, b)$;
- ☐ $g(x)$ is continuous on $[a, b]$, and the values of $g(x)$ belong to $[a, b]$ ($g(x)$ maps $[a, b]$ into itself), and $g'(x)$ exists on (a, b), and a constant k, $0 < k < 1$ exists such that $|g'(x)| \leq k$ for all $x \in (a, b)$, and there exists p such that $g(p) = p$;

1c. (5pt) In Theorem 2.5 (convergence of Newton's method), what is the significance of condition $f'(p) \neq 0$?

Theorem 2.5: Let $f(x) \in C^2[a,b]$, if $p \in [a,b]$ exists such that $f(p) = 0$, and $f'(p) \neq 0$, then there exists $\delta > 0$ such that Newton's method generates a sequence $\{p_n\}_n^\infty$ converging to p for any initial approximation $p_0 \in [p-\delta, p+\delta]$.

The significance of condition $f'(p) \neq 0$ was

- ☐ it is not a part of the theorem;
- ☐ because of this condition, the sequence $p_n = p_{n-1} - f(p_{n-1})/f'(p_{n-1})$ diverges;
- ☐ because of this condition, the derivative $g'(x) = f(x)f''(x)/[f'(x)]$ is zero;
- ☐ because of this condition, there exists interval near p where $f(x)$ is zero;
- ☐ because of this condition, function $g(x) = x - f(x)/f'(x)$ is continuous near p.

1d. (5pt) In the proof of Theorem 3.3 (error estimate in interpolation by Lagrange's Polynomial), what has to be noticed about the function

$$g(t) = f(t) - P(t) - [f(x) - P(x)]\frac{(t-x_0)(t-x_1)\cdot\ldots\cdot(t-x_n)}{(x-x_0)(x-x_1)\cdot\ldots\cdot(x-x_n)}.$$

Theorem 3.3: Suppose $x_0, x_1, \ldots, x_n$ are distinct numbers in the interval $[a,b]$ and $f(x) \in C^{n+1}[a,b]$ then, for each $x \in [a,b]$ a number $\xi(x) \in (a,b)$ (generally unknown) exists with

$$f(x) = P(x) + \frac{f^{(n+1)}(\xi(x))}{(n+1)!}(x-x_0)(x-x_1)\cdot\ldots\cdot(x-x_n),$$

where $P(x)$ is Lagrange's interpolating polynomial.

- ☐ The key was to notice that $g(t)$ is zero at $n+1$ distinct points;
- ☐ The key was to notice that $g(t)$ has no derivative at $t = x_0, x_1, \ldots, x_n$;
- ☐ The key was to notice that $g(t)$ is zero on interval $[a,b]$;
- ☐ The key was to notice that $g(t)$ is an interpolating polynomial;
- ☐ none of the above.

2. (7pt) Give the definition of linear convergence.

(13pt) Show that the sequence $p_n = \dfrac{1}{n^3}$ converges linearly to 0.

3. (7pt) Give the definition of Newton's method for solving the root finding problem.

(13pt) Give the definition of Horner's method (calculating value of a polynomial, division of a polynomial)

4. (7pt) Formulate the fixed point iteration method for finding the root of $f(x) = x^3 + x - 4$ on interval $[0, 2]$.

(13pt) Estimate the number of steps required to calculate a root within 10^{-2} accuracy. Use initial approximation $p_0 = 1$.

5. (10pt) Give definition of nth Lagrange's polynomial for interpolating function $f(x)$ at the nodes $x_0, x_1, \ldots, x_n$.

(10pt) Write interpolating polynomial of degree 2 for the function $y = \sqrt{\cos(x)}$ using nodes $x_0 = -\pi/2$, $x_1 = 0$, $x_2 = \pi/2$.