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Last Name:________________

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Math 481a Final. May 23, 2005

Attention! Please, note that this is the closed book test. You are allowed to use a graphing calculator. In multiple choice, guessing is not penalized. In other problems, please, show all important steps in you solution.

1a. (5pt) The $(n+1)$**-point formula** to approximate $f'(x)$ can be obtained by

- ☐ by differentiating the equations n times;
- ☐ by applying higher-order Taylor methods;
- ☐ applying forward-difference equation;
- ☐ applying backward-difference equation;
- ☐ differentiating n-th Lagrange polynomial approximating $f(x)$.

1b. (5pt) Let $f(x)$ be a continuous function on the interval $[a, b]$, and let the interval be subdivided into an even number n of subintervals by the nodes $x_0 = a,\ x_1,\ x_2,\ \ldots,\ x_n = b$. The quadrature formula

$$\int_a^b f(x)\mathrm{d}x \approx \frac{h}{3}\left[f(x_0) + 2\sum_{j=1}^{(n/2)-1} f(x_{2j}) + 4\sum_{j=1}^{n/2} f(x_{2j-1}) + f(x_n)\right]$$

is called

- ☐ The Composite Trapezoidal rule;
- ☐ Trapeziodal Rule;
- ☐ Simpson's Rule;
- ☐ The Adaptive Quadrature Rule;
- ☐ none of the above.

1c. (5pt) The error estimate

$$|y(t_i) - w_i| \le \frac{hM}{2L}[\mathrm{e}^{L(t_i-a)} - 1]$$

- ☐ is for the Euler Method;
- ☐ is for the Adaptive quadrature method;
- ☐ is for the Composite Simpson's method;
- ☐ is the local truncation error estimate for the Runge-Kutta method;
- ☐ is the local truncation error estimate for the Modified Euler Method.

1d. (5pt) In derivation of the Midpoint Method, which is a Runge-Kutta method or order two for solving the initial value problem $y' = f(t, y)$, $a \le t \le b$, $y(a) = \alpha$, the function

$$T^{(2)}(t, y) = f(t, y) + \frac{h}{2}\frac{\partial f}{\partial t}(t, y) + \frac{h}{2}\frac{\partial f}{\partial y}(t, y)f(t, y)$$

is being approximated by the function

- ☐ $a_1 f(t, y) + a_2 f(t + \alpha_2, y + \delta_2 f(t, y))$;
- ☐ $a_1 f(t + \alpha_1, y + \beta_1)$;
- ☐ $[f(t + \alpha_1, y + \beta_1)]^{a_1}$;
- ☐ $f(t + \alpha_1, y + \beta_1) - f(t + \alpha_2, y + \beta_2)$;
- ☐ none of the above.

2. (10pt) For the following equation, determine an interval $[0, b]$ on which the fixed-point iteration will converge

$$x = \left(\frac{\mathrm{e}^x}{3}\right)^{1/2}$$

(10pt) Give the definition of the **osculating polynomial**.

3. (5pt) Give the definition of the **natural cubic spline**.

(10pt) A free cubic spline on the interval $[1, 3]$ is given by

$$S(x) = \begin{cases} 1 + 2(x-1) + (x-1)^3, & \text{if} \quad 1 \le x < 2 \\ a + b(x-2) + c(x-2)^2 + d(x-2)^3, & \text{if} \quad 2 \le x \le 3 \end{cases}$$

Determine coefficients a, b, c, d.

4. (5pt) Write the definition of the **forward-difference formula** and **backward-difference formula** (the two-point formulas) for approximating $f'(x_0)$.

(10pt) Find error bounds for approximating $f'(x_0)$ by the forward-difference formula, if

$$f(x) = x^2 \ln x + 1, \qquad x_0 = 1,\ x_1 = 1.01$$

5. (5pt) Explain what is meant by saying that the quadrature formula

$$\int_0^1 f(x)\mathrm{d}x \approx c_1 f(0) + c_2 f'(0) + c_3 f''(0) + c_4 f'''(0)$$

has **degree of precision** three.

(10pt) Determine the constants c_1, c_2, c_3, c_4 that produce a quadrature formula

$$\int_0^1 f(x)\mathrm{d}x \approx c_1 f(0) + c_2 f'(0) + c_3 f''(0) + c_4 f'''(0)$$

that has degree of precision three.

6. Consider the initial value problem problem

$$y'(t) = 1 - (t - y)^2, \qquad 0 \le t \le \pi, \qquad y(0) = 1.$$

(5pt) Write the discrete equation resulting from the Midpoint Method, if $h = 0.05$,

Hint: The Midpoint Method:

$$w_0 = \alpha,$$
$$w_{i+1} = w_i + hf\Big(t_i + \frac{h}{2}, w_i + \frac{h}{2} f(t_i, w_i)\Big)$$

(5pt) Write the expression for the local truncation error $\tau_{i+1}(h)$,

(5pt) What is the order of the local truncation error for the Midpoint Method? For the Modified Euler Method?