Independence of Path

I-D:

Fundamental theorem of Calculus
(Newton-Leibnitz Formula)

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

only ENDS ARE important

$\Rightarrow$ CAN THERE BE AN ANALOG IN 2-D-?? (3-D?) $\Rightarrow$

ANALOGS

I-D	2,3-D
$f'(x)$ $\Rightarrow$	$\nabla f(x,y,z)$
$\int_a^b f(x)\,dx$ $\Rightarrow$	$\int_C f(x,y,z)\,ds$

line integral

How about to check $\int_C (\overline{\nabla} f(x,y,z) \cdot ???)\,ds$

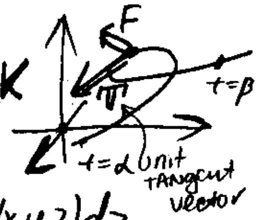
Recall line integrals: line C is given by $x = x(t)$, $y = y(t)$, $z = z(t)$, $\alpha \le t \le \beta$

$$\int_C f(x,y,z)\,ds = \int_\alpha^\beta f(x(t),y(t),z(t))\sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\,dt$$

Area of a curtain

$$\int_C \overline{F}(x,y,z)\cdot \overline{T}\,ds = \int_\alpha^\beta \overline{F(x(t),y(t),z(t))}\cdot \frac{d\overline{r}}{dt}\,dt = \int_C \overline{F}(x,y,z)\cdot d\overline{r} =$$

WORK



$$= \int_C \overline{F}(x,y,z)\cdot(dx, dy, dz) = \int_C F_x(x,y,z)\,dx + F_y(x,y,z)\,dy + F_z(x,y,z)\,dz$$

x component of $\overline{F}$

Consider:

$$\int_C \overline{\nabla} f(x,y,z)\cdot d\overline{r} = \int_\alpha^\beta \overline{\nabla} f(x(t),y(t),z(t))\cdot \frac{d\overline{r}}{dt}\,dt =$$

$$= \int_\alpha^\beta \overline{\nabla} f(x(t),y(t),z(t))\cdot(x'(t), y'(t), z'(t))\,dt = \int_\alpha^\beta \left[\frac{\partial f}{\partial x}x'(t) + \frac{\partial f}{\partial y}y'(t) + \frac{\partial f}{\partial z}z'(t)\right]dt =$$

$$= \int_\alpha^\beta \frac{d}{dt}\left[f(x(t),y(t),z(t))\right]dt = \int_\alpha^\beta \varphi'(t)\,dt = \varphi(\beta) - \varphi(\alpha)$$

$$= f(x(\beta),y(\beta),z(\beta)) - f(x(\alpha),y(\alpha),z(\alpha))$$