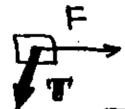
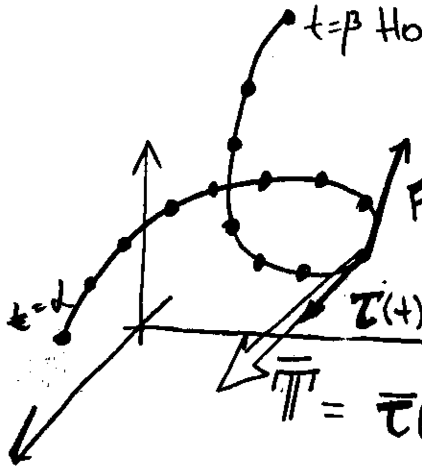
Work



THE WORK DONE BY FORCE $\overline{F}$ to move an object IN (on?) Displacement $\overline{\mathbb{T}}$ is $W = \overline{F} \cdot \overline{\mathbb{T}}$

(dot product of $\overline{F}$ and $\overline{T}$)

NO FORCE = NO WORK
NO Displacement = no work
ALSO, $F \perp \mathbb{T}$ = NO WORK



How to calculate the Work of the Force $F(x,y,z)$ to move the object along curve $C = \begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases}$

or $\overline{r}(t) = (x(t), y(t), z(t))$

$$\tau(t) = (x'(t), y'(t), z'(t)) \cdot \frac{1}{\sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}}$$

$\overline{\tau}$ is the unit tangent vector

$$\overline{\mathbb{T}} = \overline{\tau}(t)\, ds$$

ds is the displacement along the curve $\Rightarrow$

$$ds = \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\, dt$$

$$dW = \overline{F} \cdot \overline{\mathbb{T}} = \overline{F} \cdot \overline{\tau}(t)\, ds$$

$$= \overline{F}(x(t), y(t), z(t)) \cdot \overline{(x'(t), y'(t), z'(t))} \cdot \frac{1}{\sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}} \cdot \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\, dt$$

$$= \overline{F}(x(t), y(t), z(t)) \cdot \overline{(x'(t), y'(t), z'(t))} \cdot dt = \overline{F} \cdot \frac{d\overline{r}}{dt}\, dt$$

Finally

THE WORK OF the force field $F(x,y,z)$ in moving a particle along a positive oriented curve C $(\overline{r}(t) = (x(t), y(t), z(t)))$

unit tangent vector

$$W = \int_C \overline{F} \cdot \overline{\tau}\, ds = \int_\alpha^\beta \overline{F}(x(t), y(t), z(t)) \cdot \frac{d\overline{r}(t)}{dt}\, dt$$

line integrals

2-D: $\displaystyle\int_C f(x,y)\, ds = \int_\alpha^\beta f(x(t), y(t)) \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$

3-D: $\displaystyle\int_C f(x,y,z)\, ds = \int_\alpha^\beta f(x(t), y(t), z(t)) \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\, dt$