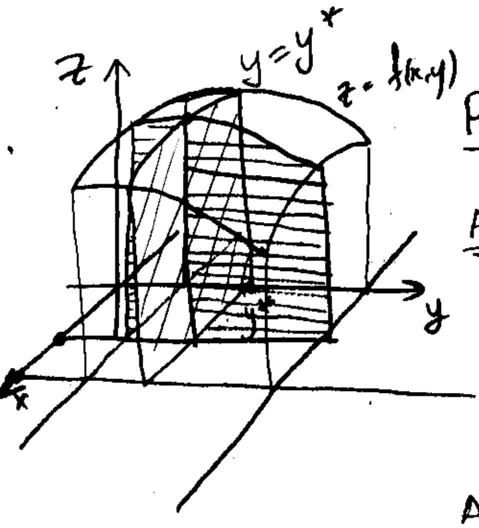
Line integral



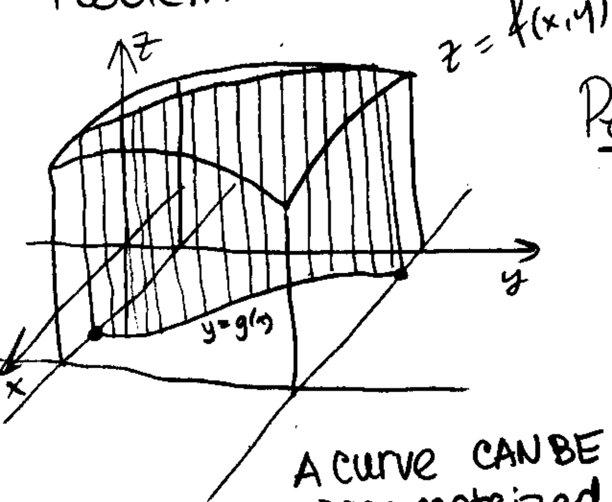
Problem 1: Area of the section by y = const

Answer: $$A_1 = \int_a^b f(x, const)\,dx$$

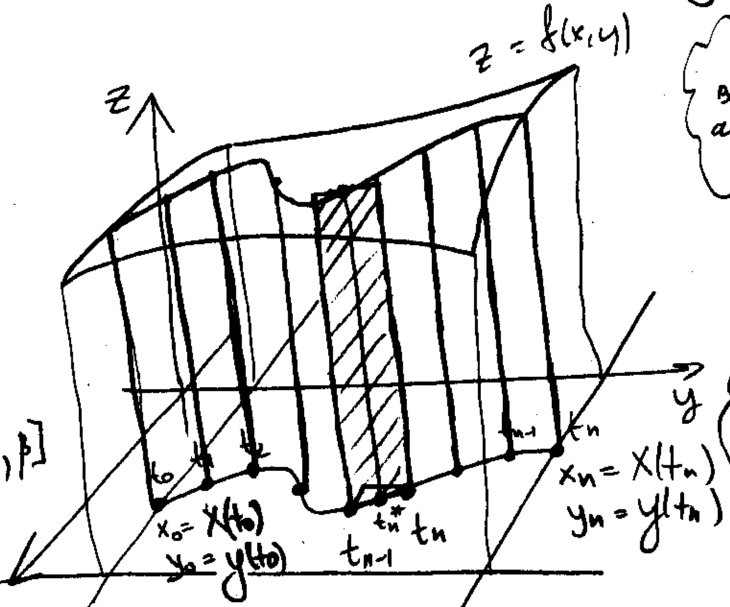
Problem 2: Area of the section by x = const

Answer: $$A_2 = \int_c^d f(const, y)\,dy$$

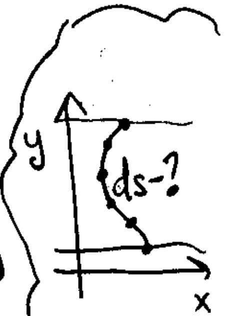
Problem 3:



Problem 3: Area of the section by $y = g(x)$



this one, for example, can be parameteized as $\begin{cases} x = t \\ y = g(t) \end{cases}$



A curve CAN BE parameteized by $\begin{cases} x = x(t) \\ y = y(t) \end{cases}$ $t \in [\alpha, \beta]$

we introduce a partitioning in t: $t_0 = \alpha, t_1, t_2, \ldots, t_n = \beta$

$$\Delta A_n = f(x(t_n^*), y(t_n^*)) \cdot \Delta S_n$$

$$A_3 = \int_C f(x,y)\,ds = \boxed{\text{we will assume that the curve is positive oriented = motion is in direction of increasing } t} =$$

$$= \lim_{\Delta t_n \to 0} \sum_i f(x(t_i^*), y(t_i^*)) \sqrt{[x'(t_i)]^2 + [y'(t_i)]^2}\,\Delta t_i$$

$$= \int_\alpha^\beta f(x(t), y(t)) \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$$

$$\Delta S_n = \sqrt{(x_n - x_{n-1})^2 + (y_n - y_{n-1})^2} =$$

$$= \sqrt{(x(t_n) - x(t_{n-1}))^2 + (y(t_n) - y(t_{n-1}))^2}$$

$$\cong \sqrt{(x'(t_n)\Delta t_n)^2 + (y'(t_n)\Delta t_n)^2}$$

$$= \sqrt{(x'(t_n))^2 + (y'(t_n))^2}\,\Delta t_n$$