OPERATORS on VECTOR FIELDS

Gradient of a scalar function

THIS vector-function we have already met

$$\overline{F}(x,y,z) = \overline{\nabla} f(x,y,z) = \left(\frac{\partial f}{\partial x}(x,y,z), \frac{\partial f}{\partial y}(x,y,z), \frac{\partial f}{\partial z}(x,y,z)\right)$$

It is, in fact, a very special case when a vector field is a gradient of a scalar field: such $\overline{F}(x,y,z) = \overline{\nabla} f$ is called the convervative field, f is its potential function

Divergence of vector-function

$$\operatorname{div} \overline{G}(x,y,z) = \frac{\partial G_1(x,y,z)}{\partial x} + \frac{\partial G_2}{\partial y}(x,y,z) + \frac{\partial G_3}{\partial z}(x,y,z)$$

Curl of vector-function

$$\operatorname{curl} \overline{G}(x,y,z) = \left(\frac{\partial G_3}{\partial y} - \frac{\partial G_2}{\partial z}, \frac{\partial G_1}{\partial z} - \frac{\partial G_3}{\partial x}, \frac{\partial G_2}{\partial x} - \frac{\partial G_1}{\partial y}\right) =$$

OTHER Definitions for curl

$$= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ G_1 & G_2 & G_3 \end{vmatrix} = \overline{\nabla} \times \overline{G}$$

∇: scalar $\to$ vector	$\operatorname{div} F = \nabla \cdot F$
div: vector $\to$ scalar	$\operatorname{curl} F = \nabla \times F$
curl: vector $\to$ vector	