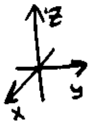
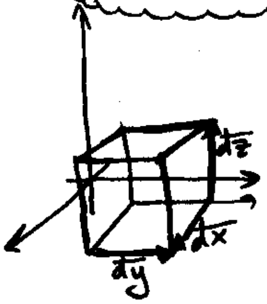
CHANGE of variables in triple integrals.

$(x,y,z) \longrightarrow (u,v,w)$

$x = x(u,v,w)$
$y = y(u,v,w)$
$z = z(u,v,w)$



Cartesian



$dV = dx\,dy\,dz$

To write the change of variables formula, the only thing we need to know is the expession for the volume element in new variables

THE SAME dV, but in new coordinates

MAY BE THIS IS NOT THE SMARTEST NATURAL PARTITIONING

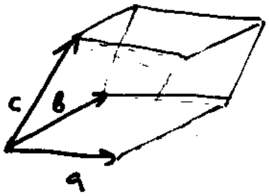
BUT, STILL, IT WORKS!

HOW TO get AN EXPrESSion for dV in new variables ?????

$\Rightarrow$ RECALL WHAT DO we Remember about calculating volumes: $\Rightarrow$

$\Rightarrow$ Here is a good formula:

The volume of a parallelepiped made out of vectors



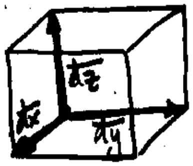
$a = (a_1, a_2, a_3)$
$b = (b_1, b_2, b_3)$
$c = (c_1, c_2, c_3)$

$$V = |a\cdot(b\times c)| = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \quad (*)$$

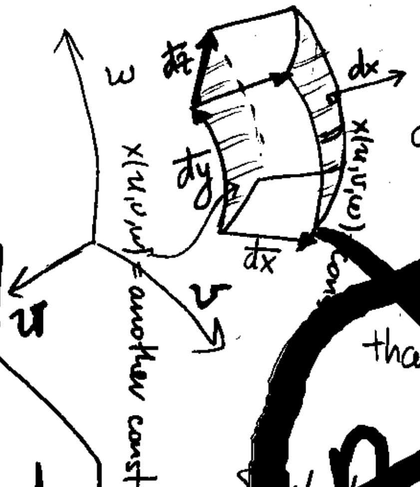
determinant of the matrix

$\Rightarrow$

IF we could express dx, dz, dy in terms of u, v, w then we could just use $(*)$



Let us go to the picture of dV In the new coordinates:



Let us find an expression for dx

dx is perpendicular to surface

~~$X(u,v,w) = $ const. In fact, $dx = \nabla X(u,v,w)$ that is ... Similarly $dy = \left(\frac{\partial y}{\partial u}, \frac{\partial y}{\partial v}, \frac{\partial x}{\partial w}\right)$, $dz = \left(\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}, \frac{\partial z}{\partial w}\right)$~~

WRONG

$$dV = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} du\,dv\,dw$$

$$\iiint_S f(x,y,z)\,dV = \iiint_S f(x(u,v,w), y(u,v,w), z(u,v,w)) \cdot \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} du\,dv\,dw$$

"in cartesian" "in new"

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