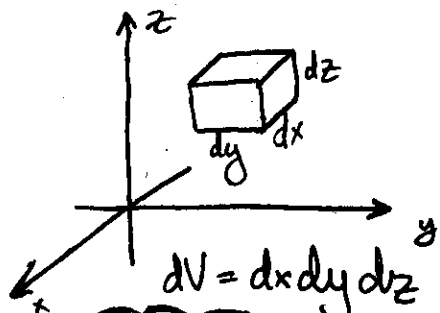
Triple integrals in spherical and cylindrical coordinates.

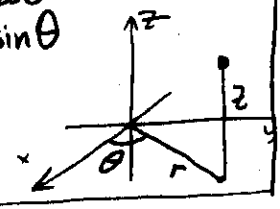
Natural volume element: CART



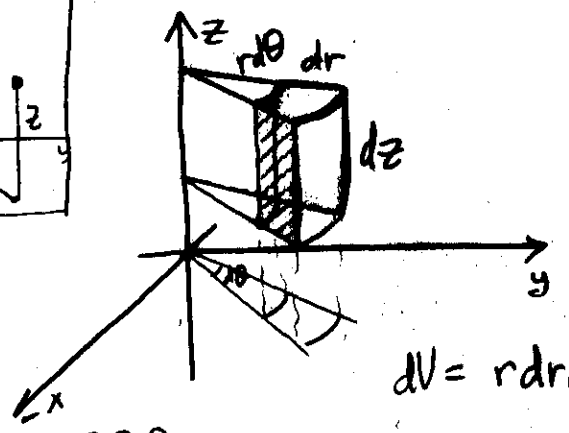
$dV = dx\,dy\,dz$

Cartesian ⇒ Cylindrical

$x = r\cos\theta$
$y = r\sin\theta$
$z = z$



Natural volume element: CYL



$dV = r\,dr\,d\theta\,dz$

Warning!

Never switch

(x^2-1) → $(r^2\cos^2\theta - 1)$

CART. CYL.

YOU DIDN'T NEED TO KNOW THAT

$y = r^2\cos^2\theta - 1$

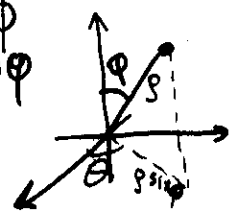
$$\iiint_S f(x,y,z)\,dV = \iiint_S f(r\cos\theta, r\sin\theta, z)\, r\,dr\,d\theta\,dz$$

"in Cartesian" "in Cylindrical"

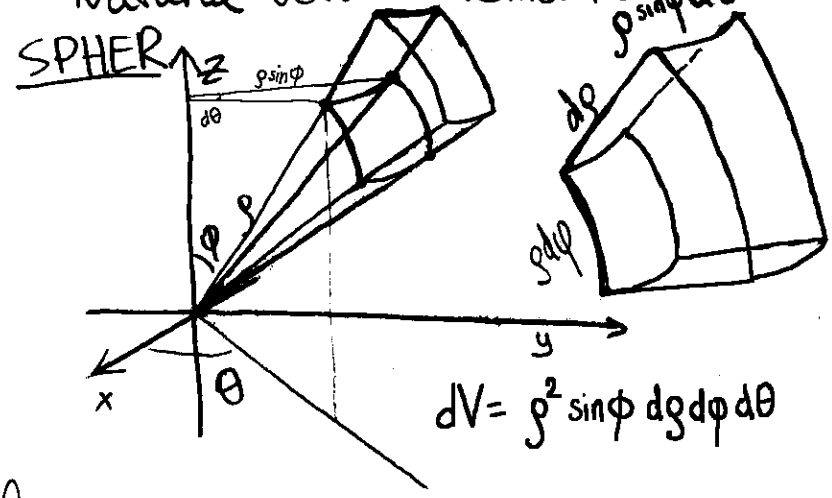
The order of $dr\,d\theta\,dz$ is not FIXED at this time

Cartesian ⇒ Spherical

$x = \rho\cos\theta\sin\phi$
$y = \rho\sin\theta\sin\phi$
$z = \rho\cos\phi$



Natural Volume Element: SPHER



$dV = \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

$$\iiint_S f(x,y,z)\,dV = \iiint_S f(\rho\cos\theta\sin\phi, \rho\sin\theta\sin\phi, \rho\cos\phi)\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

"in cartesian" "in spherical"

Again, the order of $d\rho, d\theta, d\phi$ is not Fixed