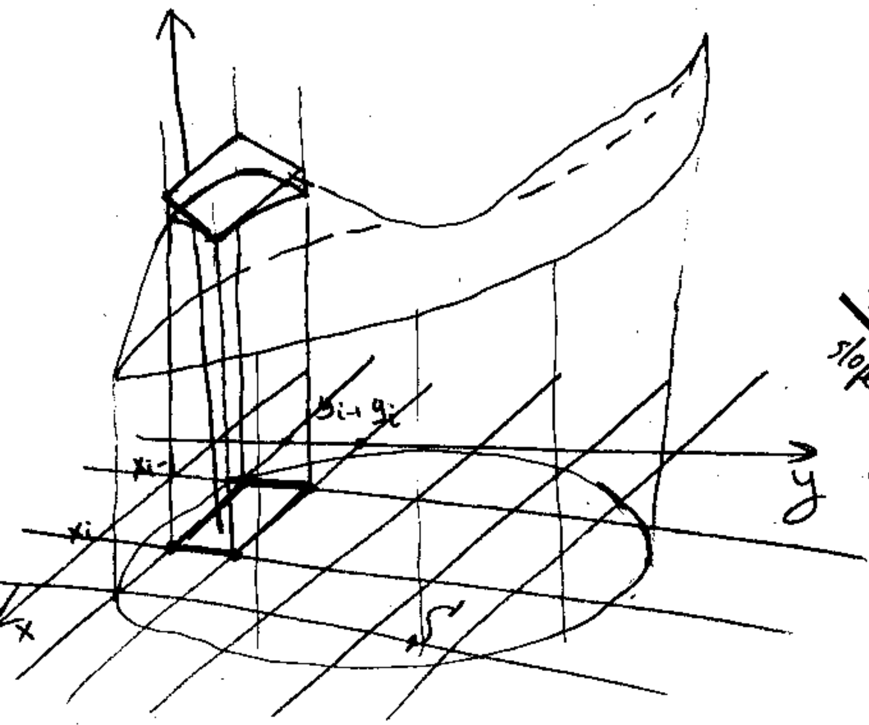
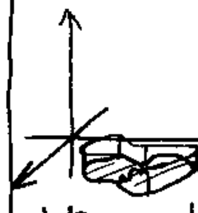
Surface Area

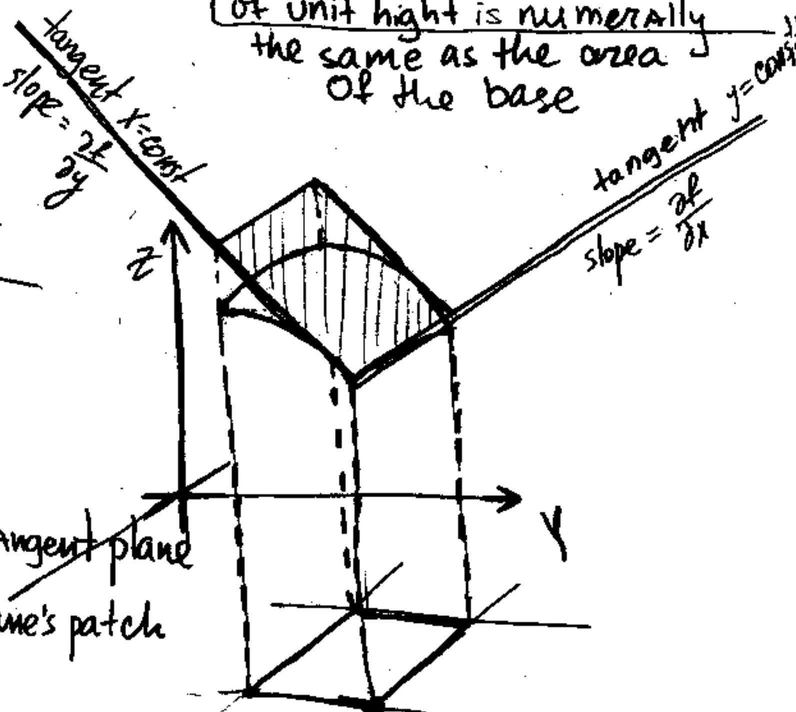


Do not mix with the area of a set S in xy-plane:



$$A=\iint_S 1\cdot dA=\iint_S dA$$

we use trivial fact that the volume of a cylinder of unit hight is numerally the same as the area of the base



Patch on the surface ≈ Patch on the tangent plane

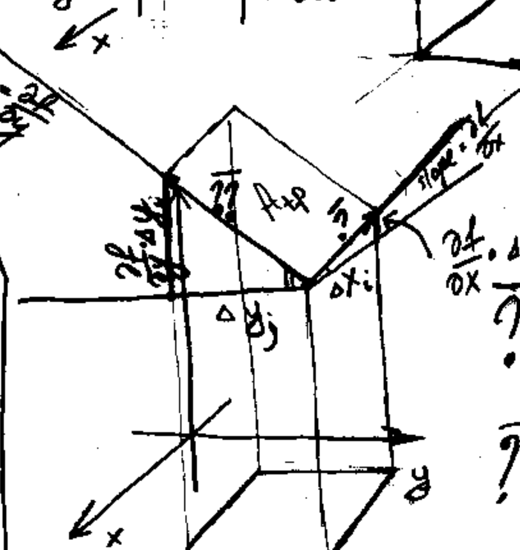
Area of the patch ≈ Area of the tangent plane's patch

Area of the surface $z=f(x,y)$ above set S is:

$$A=\lim_{\substack{\Delta x_i\to 0\\ \Delta y_j\to 0}}\sum_{ij}\sqrt{1+\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2}\,\Delta x_i\Delta y_j$$

or,

$$A=\iint_S\sqrt{1+\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2}\,dA$$



$$? = \left(\Delta x_i, 0, \frac{\partial f}{\partial x}\cdot\Delta x_i\right)$$

$$?? = \left(0, \Delta y_j, \frac{\partial f}{\partial y}\cdot\Delta y_j\right)$$

$$A_{tp} = |??\times ?| = \text{length of}\begin{pmatrix} i & j & k\\ 0 & \Delta y_j & \frac{\partial f}{\partial y}\Delta y_j\\ \Delta x_i & 0 & \frac{\partial f}{\partial x}\Delta x_i\end{pmatrix}$$

$$= \text{length of}\left(\frac{\partial f}{\partial x}\Delta y_j\Delta x_i, \frac{\partial f}{\partial y}\Delta y_j\Delta x_i, \Delta x_i\Delta y_j\right) =$$

$$= \sqrt{1+\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2}\cdot\Delta x_i\Delta y_j$$