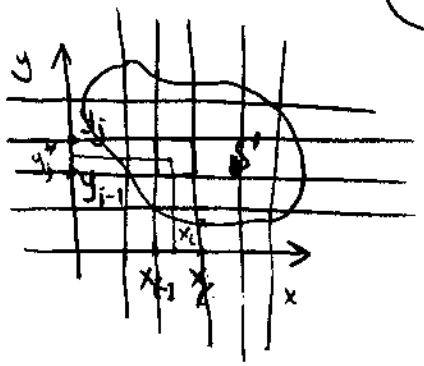
Applications of double integrals.

1 Mass of a lamina of density $\delta(x,y)$ occupying region S



$$M = \lim_{\substack{\Delta x_i \to 0 \\ \Delta y_j \to 0}} \sum_{i,j} \delta(x_i^*, y_j^*)\Delta x_i \Delta y_j = \iint_S \delta(x,y)\,dA$$

2. Center of mass: Lamina of density $\delta(x,y)$ occupying region S

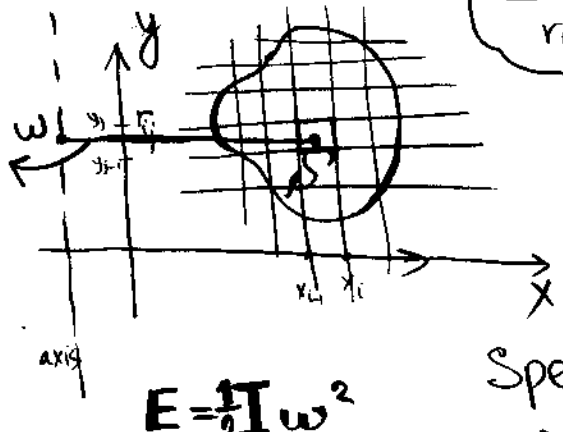
Center of mass = balance point = point around which the object spins in the free spin

Center of mass $(\bar{x}, \bar{y})$

$$\bar{x} = \lim_{\substack{\Delta x_i \to 0 \\ \Delta y_j \to 0}} \frac{\sum_{i,j} x_i^* m_{ij}}{\sum_{i,j} m_{ij}}$$

$$\bar{x} = \frac{\iint_S x\,\delta(x,y)\,dA}{\iint_S \delta(x,y)\,dA} \qquad \bar{y} = \frac{\iint_S y\,\delta(x,y)\,dA}{\iint_S \delta(x,y)\,dA}$$

3. Moment of Inertia: Lamina of density $\delta(x,y)$ occupying region S rotating around certain axis



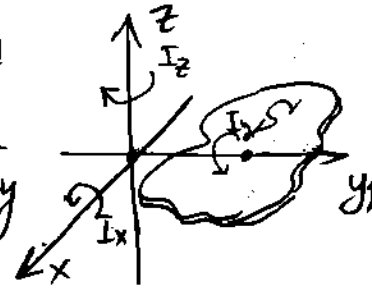
$$I_{axis} = \lim_{\substack{\Delta x_i \to 0 \\ \Delta y_j \to 0}} \sum_{i,j} r_{ij}^2 m_{ij} = \iint_S r^2(x,y)\delta(x,y)\,dA$$

$$E = \frac{1}{2} I \omega^2$$

Specifically, for rotation about coordinate axes

$$I_x = \iint_S y^2 \delta(x,y)\,dA \qquad I_y = \iint_S x^2 \delta(x,y)\,dA$$

$$I_z = \iint_S (x^2+y^2)\delta(x,y)\,dA = I_x + I_y$$



radius of gyration

$$\bar{r} = \sqrt{\frac{I}{m}}$$