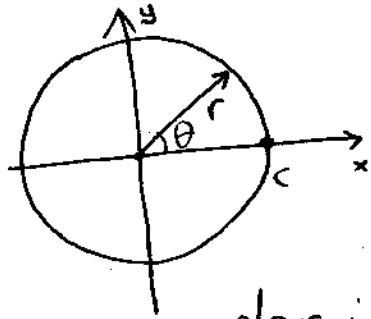
Double Integrals in polar coordinates

Equation of a circle

In cartesian

$x^2 + y^2 = C^2$



In polar

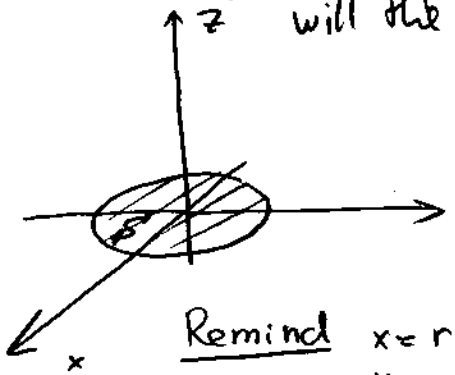
$r = C$

simpler

IF THE EQUATION IS simpler in polar coordinates will the integration be simpler also?

$\iint_S f(x,y)\,dA$

At least S is easier to express in polar coordinates



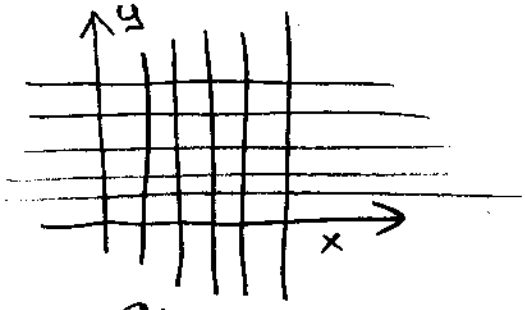
$\iint_S f(?,?)$??

simpler still ???

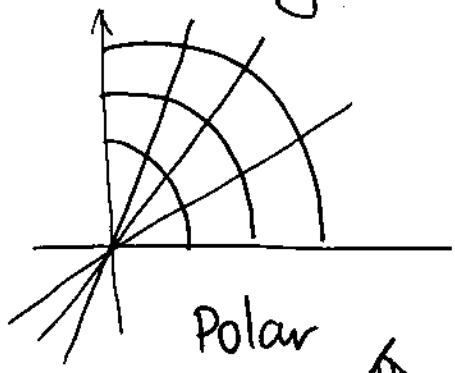
Remind $x = r\cos\theta$, $y = r\sin\theta$ $\Rightarrow$ $\iint_S f(r\cos\theta, r\sin\theta)$??

what takes place of dA ??

Natural Partitioning

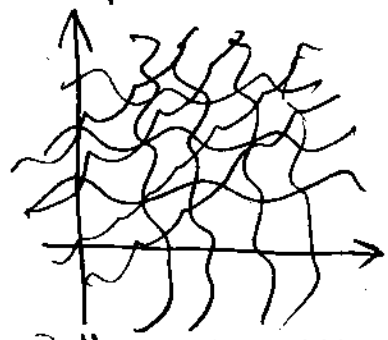


CARTesian



Polar

Stupid Partitioning



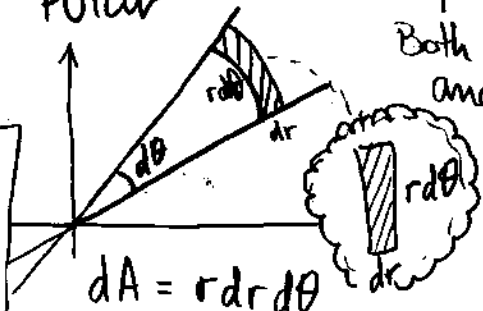
Both CARTESIAN and Polar

Compute dA:

$$\iint_S f(x,y)\,dA = \iint_S f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$$

S: "in cart." S: "in polar"

Finally:



$dA = r\,dr\,d\theta$

Similarly, we can introduce