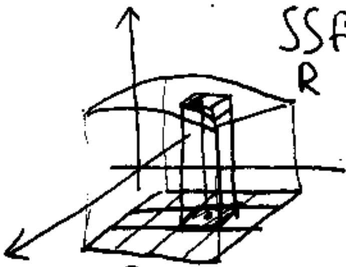
Double Integrals over Nonrectangular Regions



$$\iint_R f(x,y)\,dA = \lim_{\substack{\Delta x_i \to 0 \\ \Delta y_j \to 0}} \sum_{i,j} f(x_i^*, y_j^*)\,\Delta x_i \Delta y_j$$

Practically is evaluated by reducing to an iterated one:

$$R=[a,b]\times[c,d] \quad \int_c^d\int_a^b f(x,y)\,dx\,dy = \iint_R f(x,y)\,dA = \int_a^b\int_c^d f(x,y)\,dy\,dx$$

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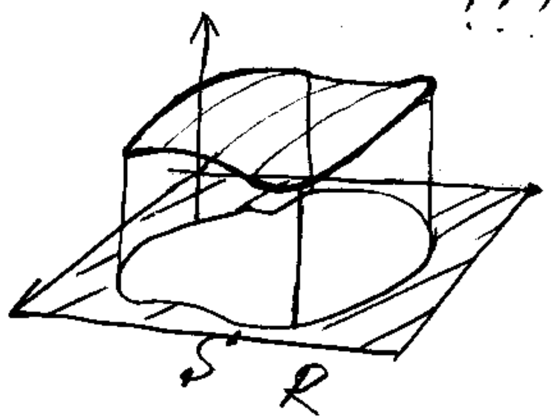
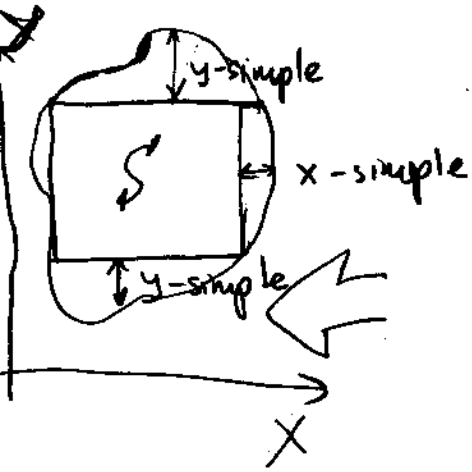


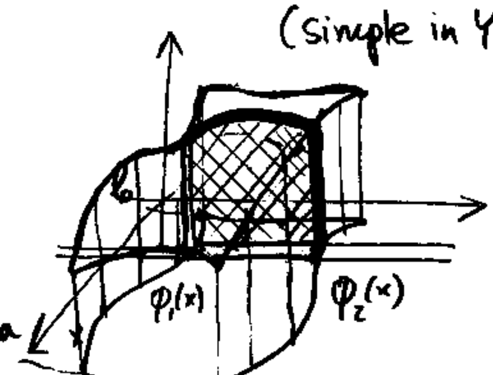
$$\iint_S f(x,y)\,dA \overset{\text{def}}{=} \iint_R \bar{f}(x,y)\,dA$$

$$\bar{f}(x,y) = \begin{cases} f(x,y), & x\in S \\ 0, & x \in R\setminus S \end{cases}$$



Practically can be computed by reducing to an iterated one:

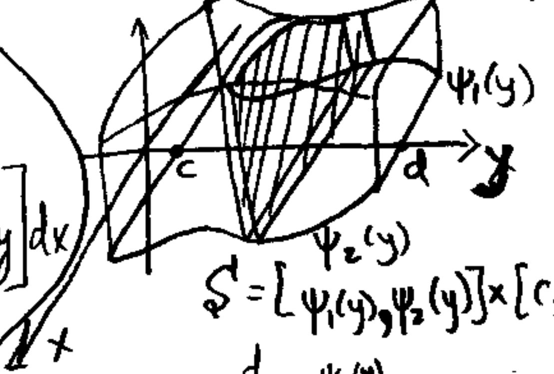
Y simple set (First integrale in Y)
(simple in Y direction)



$$\iint_S f(x,y)\,dA = \int_a^b\left[\int_{\varphi_1(x)}^{\varphi_2(x)} f(x,y)\,dy\right]dx$$

$$S=[a,b]\times[\varphi_1(x),\varphi_2(x)]$$

X-simple set (First integr. in X)
(simple in X direction)



$$S=[\psi_1(y),\psi_2(y)]\times[c,d]$$

$$\iint_S f(x,y)\,dA = \int_c^d\left[\int_{\psi_1(y)}^{\psi_2(y)} f(x,y)\,dx\right]dy$$