Double Integrals over Rectangles.

I-D

$\boxed{\text{Area under the curve}}$

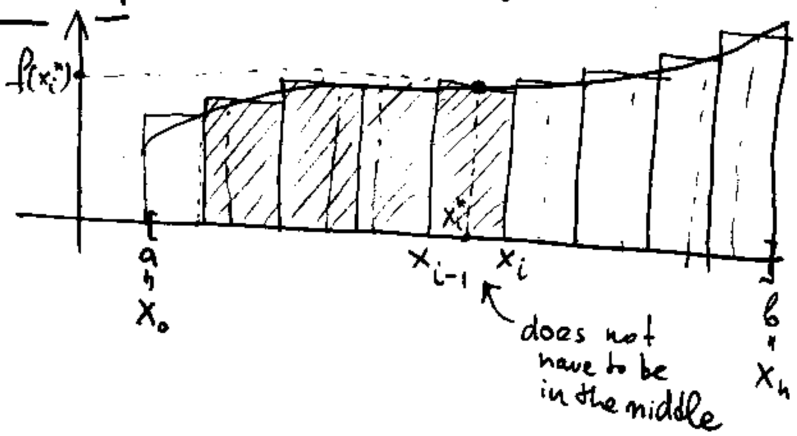
$$\int_a^b f(x)\,dx = \lim_{\Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$$

notations: $\Delta x_i = x_i - x_{i-1}$

$x_i^* \in [x_{i-1}, x_i]$

$\Delta x_i \to 0$ means

$\max\{\Delta x_1, \Delta x_2, \dots, \Delta x_n\} \to 0$



a single rect: $f(x_i^*)\cdot\Delta x_i$

all Rect: $\sum_{i=1}^{n} f(x_i^*)\Delta x_i$

In the limit: (refining the partition) $\lim_{\Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$

II-D

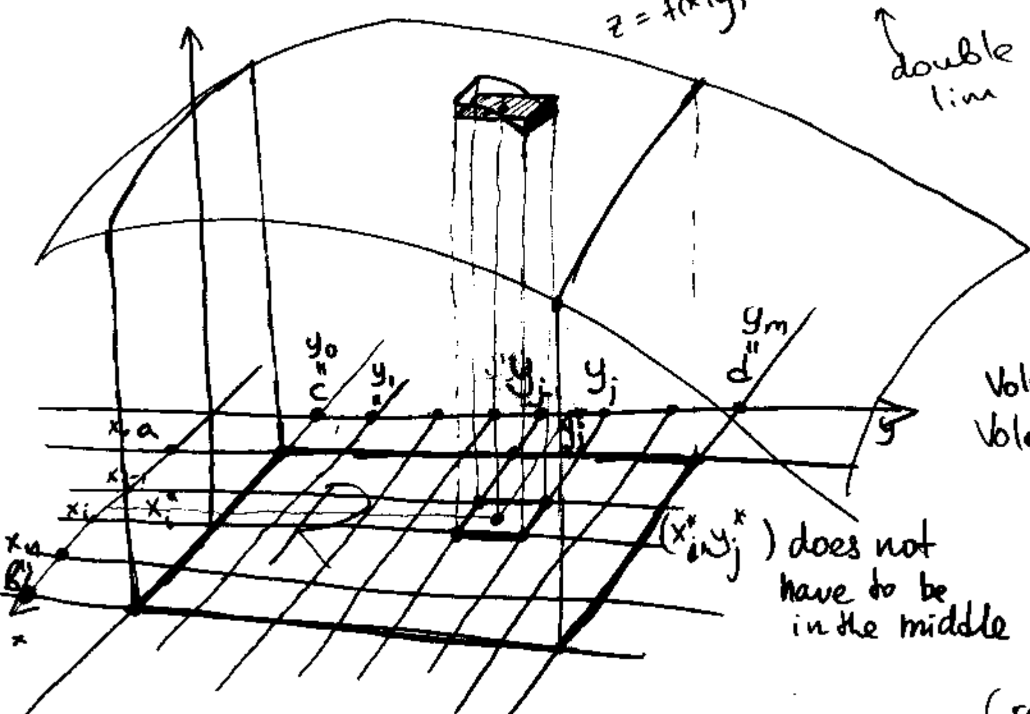
$\boxed{\text{Volume under the surface}}$

$$\iint_R f(x,y)\,dA = \lim_{\substack{\Delta x_i \to 0 \\ \Delta y_j \to 0}} \sum_{i,j} f(x_i^*, y_j^*)\underbrace{\Delta x_i \Delta y_j}$$

double lim

double sum, add all i's up, add all j's up.

Area of a little square in the partition



$R = [a,b]\times[c,d]$

Volume of single block $f(x_i^*, y_j^*)\Delta x_i \Delta y_j$

Volume overall blocks $\sum_{i,j} f(x_i^*, y_j^*)\Delta x_i \Delta y_j$

Volume under the surface (refining the partition) $\lim_{\substack{\Delta x_i \to 0 \\ \Delta y_j \to 0}} \sum_{i,j} f(x_i^*, y_j^*)\Delta x_i \Delta y_j$