Lagrange's Method (Summary)

Problem of Constrained Minimization

$f(x,y) \to \min$

$g(x,y) = 0$

Is Replaced withe the problem of minimizing

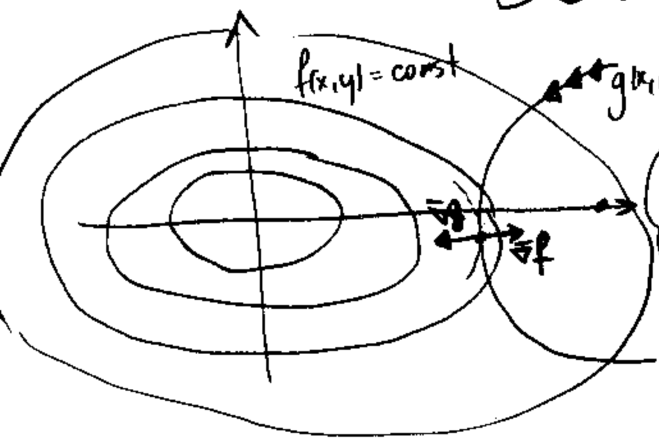
Lagrange's funct.

$$F(x,y,\lambda) = f(x,y) + \lambda \cdot g(x,y) \longrightarrow \min$$

which is solved by examining

$$\bar{\nabla} F(x,y,\lambda) = 0 \iff \begin{cases} \dfrac{\partial f}{\partial x} + \lambda \dfrac{\partial g}{\partial x} = 0 \\[2ex] \dfrac{\partial f}{\partial y} + \lambda \dfrac{\partial g}{\partial y} = 0 \\[2ex] g(x,y) = 0 \end{cases}$$

BOOK:



max or min is where $\bar{\nabla} g \parallel \bar{\nabla} f$ $\iff$ No level curves crossing

$\Rightarrow$ Lagranges eq. $\begin{cases} \bar{\nabla} f = \mu \bar{\nabla} g \\ g(x,y) = 0 \end{cases} \Rightarrow \begin{cases} \dfrac{\partial f}{\partial x} = \mu \dfrac{\partial g}{\partial x} \\[1ex] \dfrac{\partial f}{\partial y} = \mu \dfrac{\partial g}{\partial y} \\ g(x,y) = 0 \end{cases}$