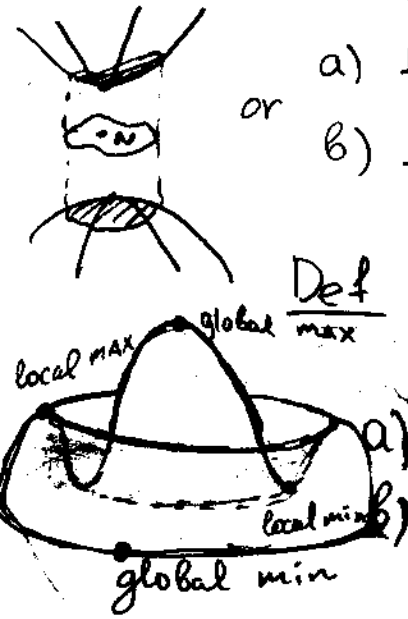
Max and minima

Def a function f has a local extremuma at p_0 if there exists a neighborhood N of the point p_0 such that either

a) $f(p) \geq f(p_0)$ for all $p \in N$ (local min at p_0)

or

b) $f(p) \leq f(p_0)$ for all $p \in N$ (local max at p_0)



Def f has a global extremuma on S at p_0 if

a) $f(p) \geq f(p_0)$ for all $p \in S$ (global min)

b) $f(p) \leq f(p_0)$ for all $p \in S$ (global max)

Where to search for extremuma? (Suspicious points)

Critical points

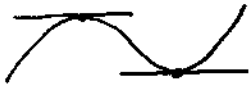
a) where $\nabla f(x) = 0$

b) where $\nabla f(x)$ D.N.E.

c) at Boundary

1-D

a)

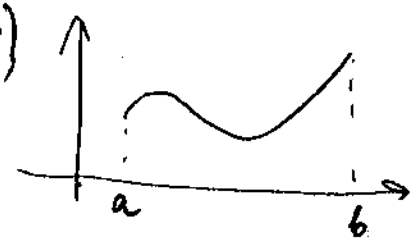


tangent line is horizontal $\Leftrightarrow$ $\Leftrightarrow f'(x_0) = 0$

b)

curve is not smooth $\Leftrightarrow$ $\Leftrightarrow f'(x_0)$ fails to exist

c)



Boundary

2-D

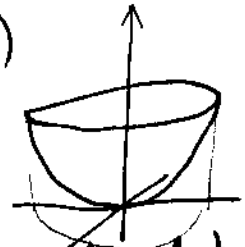
a)

tangent plane is horizontal $\Leftrightarrow$ $\Leftrightarrow \nabla f(x_0) = 0$

b)

surface is not smooth or discontinuous $\Leftrightarrow$ $\nabla f(x_0)$ fails to exist

c)



at Boundary

Min-Max Existence Thm If $f(x)$ is continuous on a closed bounded set S then f attains both a (global) max value and a (global) min value there.