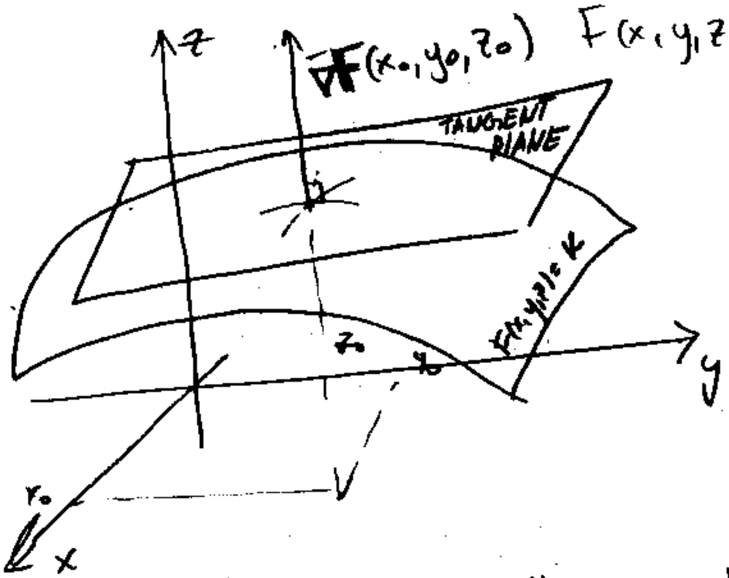
Tangent Planes

Consider a surface defined by an equation

$F(x,y,z)=k$, $k = \text{const}$

let (x_0,y_0,z_0) satisfies the eq.



Tangent plane to the surface is the plane that is tangent to all curves in the surface

(consists of all tangent lines for all possible curves)

Geometry: IF there exists a vector perpendicular to all tangent lines, this vector is perpendicular to the tangent plane

Comment: $\vec{\nabla} F(x_0,y_0,z_0)$ is such vector !!!

Now: Given the normal-vector $\nabla F(x_0,y_0,z_0)$ and the point (x_0,y_0,z_0) the equation of the tangent plane is immediate

$$A(x-x_0)+B(y-y_0)+C(z-z_0)=0$$

$$① \quad \frac{\partial F}{\partial x}(x_0,y_0,z_0)\cdot(x-x_0)+\frac{\partial F}{\partial y}(x_0,y_0,z_0)\cdot(y-y_0)+\frac{\partial F}{\partial z}(x_0,y_0,z_0)(z-z_0)=0$$

Compare this to tangent to the graph of $z=f(x,y)$ formula:

$$② \quad z=f(x_0,y_0)+\frac{\partial f}{\partial x}(x-x_0)+\frac{\partial f}{\partial y}(y-y_0)$$

Indeed, ② Follows From ① By the substitution

$$F(x,y,z)=z-f(x,y)$$

(this way eq $z-f(x,y)=0$ turns into $F(x,y,z)=0$)