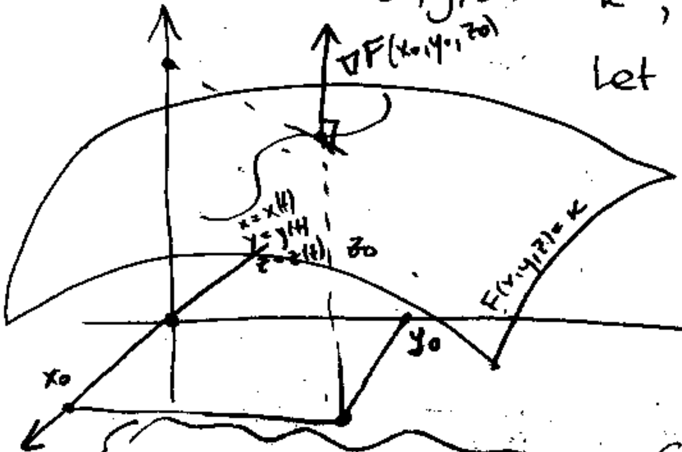
Tangent Planes

Consider a surface defined by an equation

$$F(x,y,z) = k\,, \quad k \text{ - constant}$$



Let (x_0,y_0,z_0) satisfies the eq.

Prove that...

$\overline{\nabla} F_{(x_0,y_0,z_0)}$ is perpendicular to the surface

Perpend to the curve = perpend. to the tangent to the curve.

$\overline{\nabla} F(x_0,y_0,z_0)$ is perpendicular to the surface means that $\overline{\nabla} F(x_0,y_0,z_0)$ is perpendicular to any curve that lies on the surface

OK, let's see...

Consider any curve that lies on the surface and passes through the point (x_0,y_0,z_0). Let equation of this curve is given parametrically by $\begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases}$

Notice that we use $x = x(t)$ instead of usual $x = h(t)$ to save letters. Indeed, there is no fundamental reason why we can not write $x(t)$...

Plug this into the eq:

$$F(x(t), y(t), z(t)) = k \quad (\text{for all } t\text{, since the curve lies on the surface})$$

Differentiate w.r. to t...

$$\frac{\partial F}{\partial x}\frac{dx}{dt} + \frac{\partial F}{\partial y}\frac{dy}{dt} + \frac{\partial F}{\partial z}\frac{dz}{dt} = \overline{\nabla} F \cdot \frac{d\overline{r}}{dt} = 0\,, \text{ or } \overline{\nabla} F \perp \frac{d\overline{r}}{dt}$$

Since $\frac{d\overline{r}}{dt}$ is the tangent vector to the curve $\Rightarrow$ $\overline{\nabla} F$ is perpendicular to the curve

Since the curve $\begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases}$ is any curve $\Rightarrow$ $\overline{\nabla} F$ is perpend to the surface