THE CHAIN RULE

1-D

$$\left[f(g(x))\right]' = f'(g(x))\,g'(x)$$

$$\boxed{g(x) \text{ is differentiable} \iff g(x+h) - g(x) = g'(x)h + o(h)}$$

Proof If $f(x)$ and $g(x)$ are differentiable $\Rightarrow$

$$f(g(x+h)) - f(g(x)) = f'(g(x))\left[g(x+h) - g(x)\right] + o\big(g(x+h) - g(x)\big)$$

$$= f'(g(x))\left[g'(x)h + o_g(h)\right] + o\big(g(x+h) - g(x)\big)$$

$$= \underbrace{f'(g(x))\,g'(x)}_{\left[f(g(x))\right]'} \cdot h + o_{f(g)}(h)$$

Differentiability of $\left[f(g(x))\right]$

$$\frac{d}{dx}\downarrow \; f(g(x)) \;\downarrow\; g(x) \;\downarrow\; f'(g(x))\,g'(x)$$

2-D

$$\frac{\partial}{\partial t}\downarrow \; f(x(t), y(t))$$

$$x(t) \qquad y(t)$$

$$\downarrow \qquad \downarrow$$

$$f_x(x,y)\frac{dx}{dt} \;\;?\;\; + \; f_y(x,y)\frac{dy}{dt}$$

Differentiability at (x_0, y_0)

$$f(x,y) - f(x_0,y_0) =$$

$$= \underbrace{f_x(x_0,y_0)}_{\text{"sensitivity" for a change in } x}\overbrace{(x - x_0)}^{\text{CHANGE in } x} + f_y(x_0,y_0)\overbrace{(y - y_0)}^{\text{CHANGE in } y} + o(x - x_0, y - y_0)$$

"o-little"