

THE CHAIN RULE

1-D

$$[f(g(x))]' = f'(g(x))g'(x)$$

$$g(x) \text{ is differentiable } \Leftrightarrow g(x+h) - g(x) = g'(x)h + o(h)$$

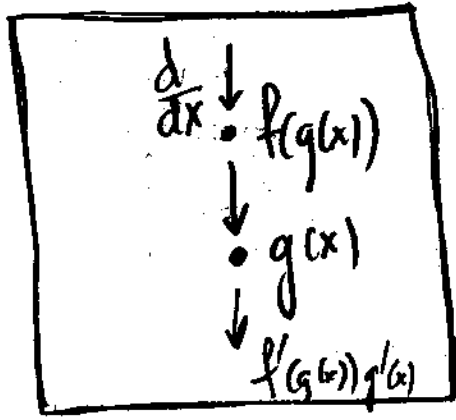
Proof If $f(x)$ and $g(x)$ are differentiable \Rightarrow

$$f(g(x+h)) - f(g(x)) = f'(g(x))[g(x+h) - g(x)] + o(g(x+h) - g(x))$$

$$= f'(g(x))[g'(x)h + o_g(h)] + o(g(x+h) - g(x))$$

$$= \underbrace{f'(g(x))g'(x)}_{[f(g(x))]' } \cdot h + o_{f \circ g}(h)$$

Differentiability at $[f(g(x))]$



2-D

Differentiability at (x_0, y_0)

$$f(x, y) - f(x_0, y_0) = \underbrace{f_x(x_0, y_0)(x - x_0)}_{\text{"sensitivity" for a change in } x} + \underbrace{f_y(x_0, y_0)(y - y_0)}_{\text{"change in } y} + o(x - x_0, y - y_0)$$

"sensitivity" for a change in x

"o-little"

