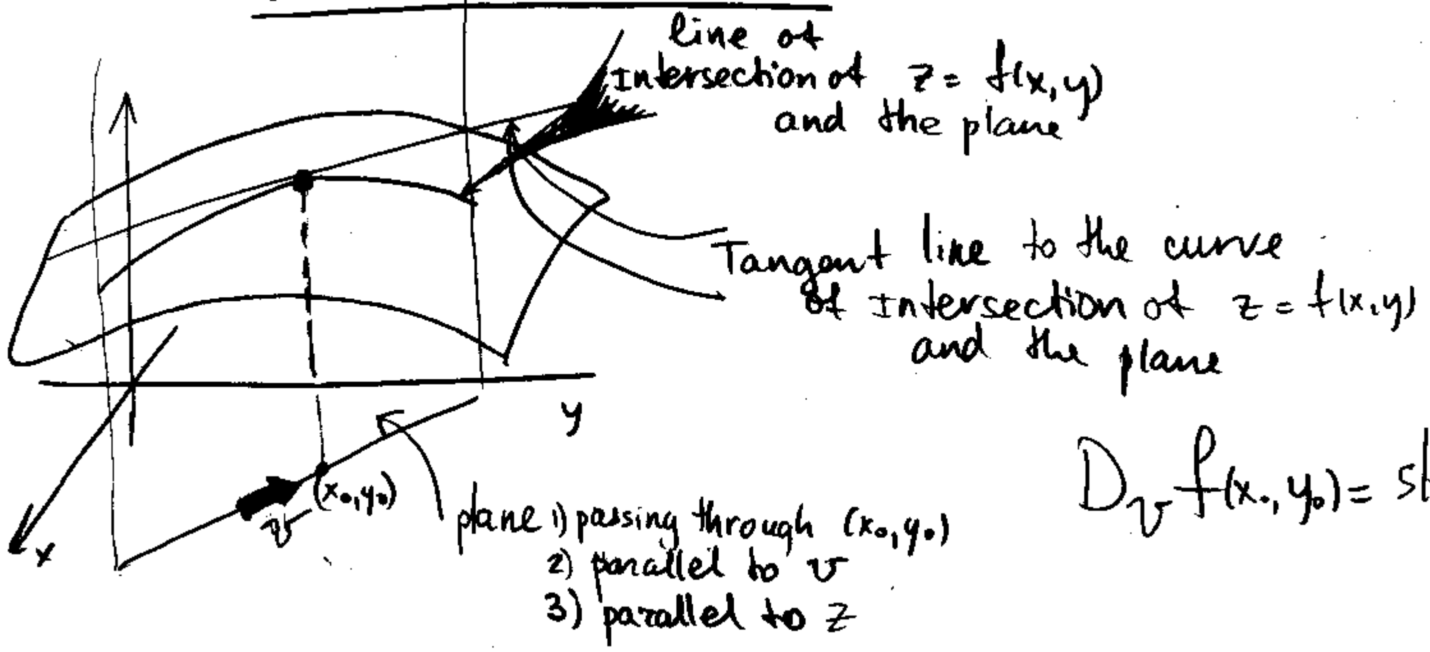
Geometrical Illustration:



$$D_v f(x_0, y_0) = \text{slope}$$

Formula for $D_v f(x,y)$

IF $f(x,y)$ is differentiable at (x,y) then $D_v f(x,y) = \frac{\partial f}{\partial x}(x,y) v_x + \frac{\partial f}{\partial y}(x,y) v_y$

Proof

Differentiable $\Leftrightarrow$

$f(x+h_x, y+h_y) = f(x,y) + \frac{\partial f}{\partial x} h_x + \frac{\partial f}{\partial y} h_y + o(h_x, h_y)$ $\Leftrightarrow$

$f(x+h_x, y+h_y) - f(x,y) = \frac{\partial f}{\partial x} h_x + \frac{\partial f}{\partial y} h_y + O(h)$

plug $h_x = v_x t$, $h_y = v_y t$

Divide by t

$$\frac{f(x+v_x t, y+v_y t) - f(x,y)}{t} = \frac{\partial f}{\partial x}\frac{v_x t}{t} + \frac{\partial f}{\partial y}\frac{v_y t}{t} + \frac{O(t)}{t}$$

As $t \to 0$

$$D_v f(x,y) = \frac{\partial f}{\partial x} v_x + \frac{\partial f}{\partial y} v_y$$

IN VECTOR NOTATIONS

$$\boxed{D_v f(x,y) = \overline{\nabla} f \cdot \overline{v}}$$

Directional derivative = Rate of change in direction v

Q: IN WHICH DIRECTION Rate of change is the fastest/slowest

A: $\overline{v} = \overline{\nabla} f$ is the direction of fastest increase

$\overline{v} = -\overline{\nabla} f$ is the direction of fastest decrease

$\overline{v} = (\nabla f)^{\perp}$ (that is $\overline{v} \perp \overline{\nabla} f$) is the direction of NO CHANGE